

Index of elliptic operators associated with diffeomorphisms of manifolds and uniformization

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Abstract

Let M be a smooth manifold and $g : M \rightarrow M$ be a diffeomorphism. We consider elliptic theory for operators of the form

$$D = \sum_k D_k T^k : C^\infty(M) \longrightarrow C^\infty(M).$$

Here $Tu(x) = u(g(x))$ is the shift operator along the orbits of g , D_k are pseudodifferential operators (ψ DO) on M , and the sum is assumed to be finite. We obtain an index formula for such operators in terms of topological invariants of the manifold and of the symbol of the operator using a new approach called *pseudodifferential uniformization*. The idea of this approach is to replace the operator D , which is essentially non-local, by an elliptic pseudodifferential operator with the same index and then apply the celebrated Atiyah–Singer formula. We note here that the symbol of operator D is an element of the crossed product $C^\infty(S^*M) \rtimes \mathbb{Z}$ of the algebra of functions on the cosphere bundle by the action of the group \mathbb{Z} . Therefore, the final index formula is naturally formulated in terms of equivariant characteristic classes in cyclic cohomology of the crossed product.

AMS Classification: 58J20.