On permutable fuzzy subgroups

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Abstract

A fuzzy subgroup μ of a fuzzy group γ on a group G is said to be permutable in γ if $\lambda \odot \mu = \mu \odot \lambda$ for every fuzzy subgroup λ of γ. Here $\alpha \odot \beta$ stands for the product of two fuzzy groups α and β on G, that is $(\alpha \odot \beta)(x) = \bigvee_{y,z \in G, yz=x} (\alpha(y) \bigwedge \beta(z))$. It is worth mentioning that in general, the product of two fuzzy subgroups is not a fuzzy subgroup. Actually, the product $\alpha \odot \beta$ is a fuzzy subgroup if and only if the fuzzy subgroups α, β are permute.

In this poster, largely extending some previous results, we characterize the permutability of fuzzy subgroups in terms of the level and support subgroups. We obtained these results emphasizing the role of the characteristic functions of elements of a group. We also show the remarkable fact that the (abstract) subgroups of a group having a fuzzy group whose fuzzy subgroups are permutable are permutable as well.

Note that this theorem is a significant generalization of the results of N. Ajmal and K.V. Thomas K.V. obtained in their article Quasinormality and fuzzy subgroups. Fuzzy Sets and Systems 58(1993), 217-225.

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