General Fractional Calculus and Evolution Equations

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Abstract

We develop a kind of fractional calculus and theory of relaxation and diffusion equations associated with operators in the time variable, of the form $(\mathbb{D}_{(k)}u)(t) = \frac{d}{dt}\int\limits_0^t k(t-\tau)u(\tau)\,d\tau - k(t)u(0)$ where k is a nonnegative locally integrable function. In particular, we find conditions on k, under which the operator $\mathbb{D}_{(k)}$ possesses a right inverse (a kind of a fractional integral) and produces, as a kind of a fractional derivative, equations of evolution type. Our results are based on the theory of complete Bernstein functions. The solution of the Cauchy problem for the relaxation equation $\mathbb{D}_{(k)}u = -\lambda u$, $\lambda > 0$, proved to be (under some conditions upon k) continuous on $[(0,\infty)$ and completely monotone, appears in the description by Meerschaert, Nane, and Vellaisamy of the process $N(E(t))$ as a renewal process. Here $N(t)$ is the Poisson process of intensity λ, $E(t)$ is an inverse subordinator.

AMS Classification: Primary 26A33, 34A08, 35R11; Secondary 60K05.