# Degenerate Elliptic Problems with mixed boundary conditions.

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**Abstract**

In this work we study the model $L_p(u) = q(x)$ where, for $p = 2$, we have $L_p(u) = \operatorname{div}(\mathcal{K}(x)\nabla u)$ with mixed (Dirichlet and Neumann) boundary conditions; for $1 < p$, $p \neq 2$ $L_p(u) = \operatorname{div}(\mathcal{K}(x)|\nabla u|^p \nabla u)$ with Dirichlet boundary condition. The non-negative-upper bounded function $\mathcal{K}$ may vanish in a subdomain $\Omega'$ of $\Omega \subset \mathbb{R}^2$, bounded domain of the problem. We use Hilbert methods to find the nontrivial solution for the case $p = 2$, and variational methods otherwise. As an application, we use $\mathbf{v} = \mathcal{K}\nabla u$ as Darcy's velocity associated with the transport-diffusion equation $\frac{\partial c}{\partial t} + \nabla \cdot (vc - D(v)\nabla c) = \widetilde{c}q,\ (x,t) \in \Omega \times [0,T]$, with inicial condition $c(x,0) = c_0$ and boundary condition upon $D$, its diffusion-dispersion tensor, to solve a system of incompressible miscible displacement with barrier. In this model $\mathcal{K}$ represents the permeability of the soil, $q$ the volumetric external flow rate per unit volume; $\widetilde{c}$ the specified concentration of solvent in the injection well ($q > 0$) and the resident concentration in the producer ($q < 0$).

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