

Noncommutative pfaffians associated with $U(\mathfrak{o}_N)$ are elements of the universal enveloping algebra defined as follows.

Let $F_{ij} = E_{ij} - E_{-j-i}$ be generators of $U(\mathfrak{o}_N)$ in the split realization, $i, j = -n, \dots, 0, \dots, n$ in the case \mathfrak{o}_{2n+1} and $i, j = -n, \dots, \widehat{0}, \dots, n$ in the case \mathfrak{o}_{2n} .

Let $\Phi = (\Phi_{ij})$, $i, j = 1, \dots, 2n$ be a skew-symmetric $2n \times 2n$ -matrix, whose matrix entries belong to a noncommutative ring. The noncommutative pfaffian of Φ is defined by the formulae

$$Pf\Phi = \frac{1}{n!2^n} \sum_{\sigma \in S_{2n}} (-1)^\sigma \Phi_{\sigma(1)\sigma(2)} \dots \Phi_{\sigma(2n-1)\sigma(2n)},$$

Here σ is a permutation of the set $\{1, \dots, 2n\}$.

For $I \subset \{-n, \dots, n\}$ we consider the matrix $F_I = (F_{-ij})_{-i, j \in I}$ and its pfaffians.

In the case $N = 2n + 1$ it is proved that some of these pfaffians, $PfF_{\widehat{-n}} = PfF_{-n+1, \dots, n}$ and $PfF_{\widehat{n}} = \{-n, \dots, n-1\}$, act on the space of \mathfrak{o}_{2n-1} -highest vectors of a \mathfrak{o}_{2n+1} -representation.

There exist the Mickelsson-Zhelobenko algebra of raising operators $Z(\mathfrak{o}_{2n+1}, \mathfrak{o}_{2n-1})$ which naturally acts on this space. We find explicitly an element of the Mickelsson-Zhelobenko algebra, which acts on this space in the same way as the pfaffian $PfF_{\widehat{n}}$. The Mickelsson-Zhelobenko algebra plays a crucial role in the construction of the Gelfand-Tsetlin-Molev base of a \mathfrak{o}_{2n+1} -representation. The Gelfand-Tsetlin-Molev base is a base of a \mathfrak{o}_{2n+1} -representation whose construction is similar to the construction of the Gelfand-Tsetlin base but which is based on restrictions $\mathfrak{o}_{2n+1} \downarrow \mathfrak{o}_{2n-1}$.

As a byproduct of these results calculations above we find explicit formulae for the action of the pfaffian $PfF_{\widehat{n}}$ in the Gelfand-Tsetlin-Molev base.

The obtained results are applied to the problem of classification of states of the five-dimensional quasi-spin. We show how one can use pfaffians $PfF_{\widehat{2}}$, $PfF_{\widehat{-2}}$ to construct an additional quantum number using which together with naturally existing quantum numbers one can classify states of a five-dimensional quasispin. The pfaffians act as creation operators of the new quantum number.