Stanley–Reisner rings of spherical nerve-complexes

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Abstract

There exists a construction which associates a simplicial complex K_P to each convex polytope P. For a simple polytope P the complex K_P coincides with the boundary ∂P^* of a polar dual polytope. In this case K_P is a simplicial sphere and its Stanley–Reisner ring $\mathbf{k}[K_P]$ is known to be Cohen–Macaulay. The global problem is to describe the properties of a simplicial complex K_P and its Stanley–Reisner ring for general convex polytope P. We develop a method to find the depth of $\mathbf{k}[K]$ for any simplicial complex K. This method is based on Hochster formula for bi-graded Betti numbers of a complex K and simple topological arguments involving Mayer–Vietoris exact sequence for simplicial cohomology. In the presentation we apply this method to the complex K_P and get an equality

$$\operatorname{depth} \mathbf{k}[K_P] = \dim P$$

for each convex polytope P.

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