Topological Obstructions to Totally Skew Embeddings

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Abstract

Affine subspaces $U_1, \ldots, U_l$ of $\mathbb{R}^N$ are called skew if their affine span has dimension $\dim(U_1) + \cdots + \dim(U_l) + l - 1$. An embedding $f : M^n \to \mathbb{R}^N$ of a smooth manifold is called *totally skew* if for each two distinct points $x, y \in M^n$ the affine subspaces $df(T_x M)$ and $df(T_y M)$ of $\mathbb{R}^N$ are skew. We study the invariant $N(M^n)$ defined as the smallest dimension N such that there exists a totally skew embedding of a smooth manifold M^n in $\mathbb{R}^N$. This problem is naturally related to the question of estimating the geometric dimension of the stable normal bundle of the configuration space $F_2(M^n)$ of ordered pairs of distinct points in M^n. We demonstrate that in a number of interesting cases the lower bounds on $N(M^n)$ obtained by this method are quite accurate and very close to the best known general upper bound $N(M^n) \leq 4n + 1$ established by Ghomi and Tabachnikov. We also provide some evidence for the conjecture that for every n-dimensional, compact smooth manifold M^n $(n > 1)$,

$$N(M^n) \leq 4n - 2\alpha(n) + 1.$$

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