

The best exponent in Markov inequality

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Abstract

Let $E \subset \mathbb{C}^N$ be a compact set preserving Markov inequality and $m(E)$ be its best exponent, i.e. $m(E)$ is the infimum of all possible exponents in this inequality on E . By Cauchy's integral formula and the Bernatein-Walsh-Siciak inequality, we can easily obtain $\alpha(E) \leq \frac{1}{m(E)}$ where $\alpha(E)$ is the best exponent in Hölder continuity property of the (pluri)complex Green function (with pole at infinity) of E . It seems that $\alpha(E) = \frac{1}{m(E)}$ for all compact sets E but it is still an open problem.

The exact values of the Markov exponent have been found for only few sets. We list these sets and we give some new examples. Moreover, we answer the following questions:

- Does the Markov inequality on E hold with the exponent $m(E)$?
- Does an upper bound for $\sup m(E)$ exist, where the supremum is taken over all sets E preserving Markov inequality?
- How does the Markov exponent of E change under a holomorphic deformation of E ?

Certain answers can be somewhat surprising.

Finally, we give simple proofs of the classical Bernstein and Markov inequalities with the exponents 1 and 2 respectively but with the constant $2e$ instead of 1.

AMS Classification: Primary 41A17; Secondary 41A44.