Large time behavior for 1D fully parabolic Keller-Segel system. Regularity for 3D axially symmetric flux obstacle problem in a cylinder.

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Abstract

1. The Keller-Segel model describes the motion of small organisms under the chemotactic forces. For the 1D fully parabolic Keller-Segel system with nonlinear diffusion a we show that:

   (i) It possesses classical global-in-time solutions, provided the nonlinear diffusion is equal to $\frac{1}{(1+u)^\alpha}$, $\alpha < 1$, independently of the volume of the initial data; in the critical case, i.e. for $\alpha = 1$, the same result holds for initial masses smaller than a prescribed constant.

   (ii) For some initial data, the solution blows up in a finite time for any nonlinear diffusion integrable at infinity.

2. For a Navier-Stokes flux model for a viscous fluid in domain Ω: a cylinder with an obstacle, under boundary slip conditions, we show:

   (i) Global-in-time existence of weak solutions.

   (ii) Existence of strong solutions in $W^{2,1}_{\frac{5}{2}}(\Omega^T)$ under the axial symmetry assumptions on domain, data and velocity field and regularity assumptions on initial forces and fluxes in weighted (by the distance from the axis) Sobolev spaces as well as smallness assumption of $\frac{\gamma}{\nu} - 2K$ in L^∞ norm (K denotes curvature of the obstacle, nu is the viscosity coefficient, γ is the friction coefficient between fluid and the obstacle).

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