Variational Models for Exemplar-Based Inpainting

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6th European Congress of Mathematics
Kraków
July 2-7, 2012

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Coauthors:

- P. Arias, V.C., G. Facciolo, Analysis of a Variational Model for Exemplar-Based Image Inpainting. SIAM MMS 2011.
- P. Arias, V.C., G. Facciolo, G. Sapiro, A Variational Framework for Exemplar-Based Image Inpainting. EMMCVPR 2009 and IJCV 2011.
- A. Hervieu, N. Papadakis, A. Bugeau, P. Gargallo, V. C. Stereoscopic Image Inpainting Using Scene Geometry. IEEE ICME, 2011.
- Y. Liu, V. C., Exemplar-based Image Inpainting using Multiscale Graph Cuts. Preprint 2012.
- G. Facciolo, R. Sadek, A. Bugeau, and V. C. Temporally consistent gradient domain video editing. EMMCVPR 2011.
- R. Sadek, G. Facciolo, P.Arias, and V. C. Temporally consistent gradient domain video editing. Preprint, 2012.

Inpainting: Examples

A couple of examples:

- An illustrative example of (more automatic) image inpainting.
- A simple example in a realistic context.

Our purpose is to explain them

Inpainting: An example

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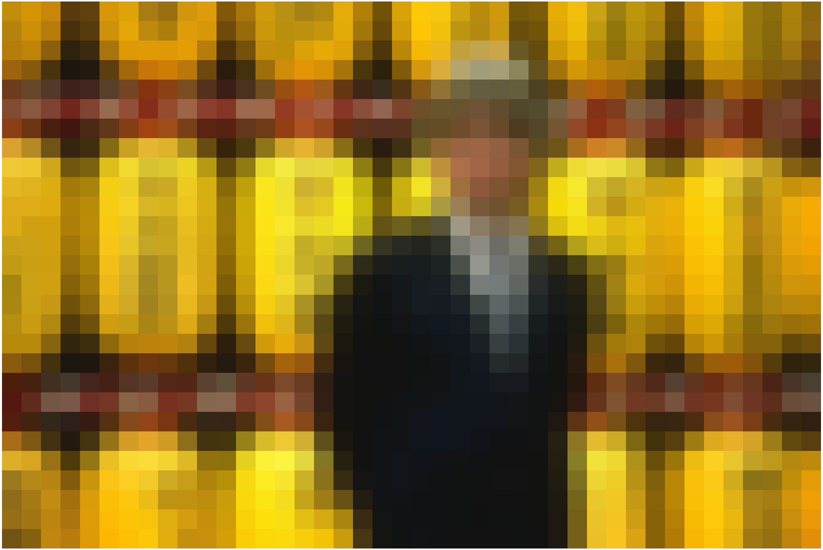

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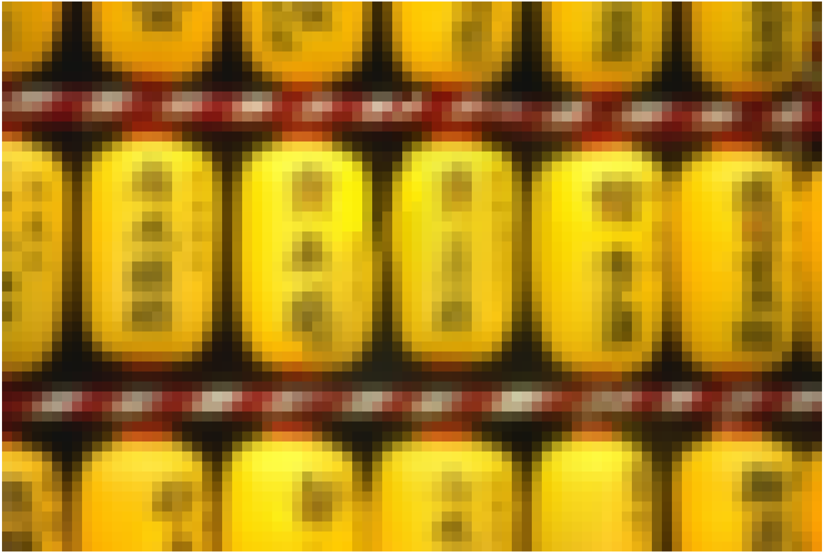

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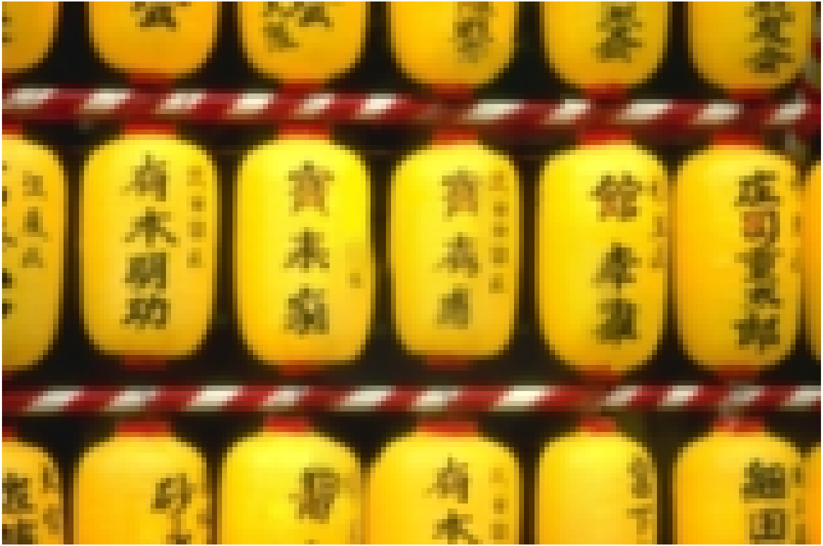

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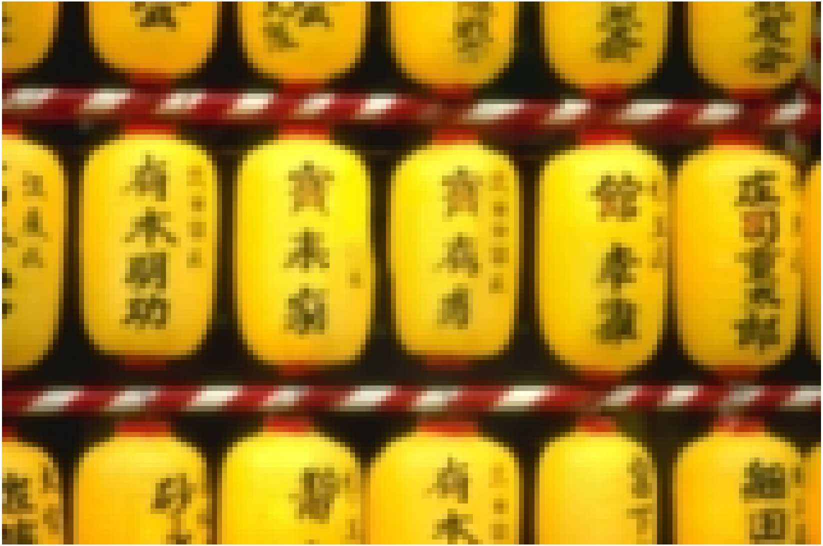

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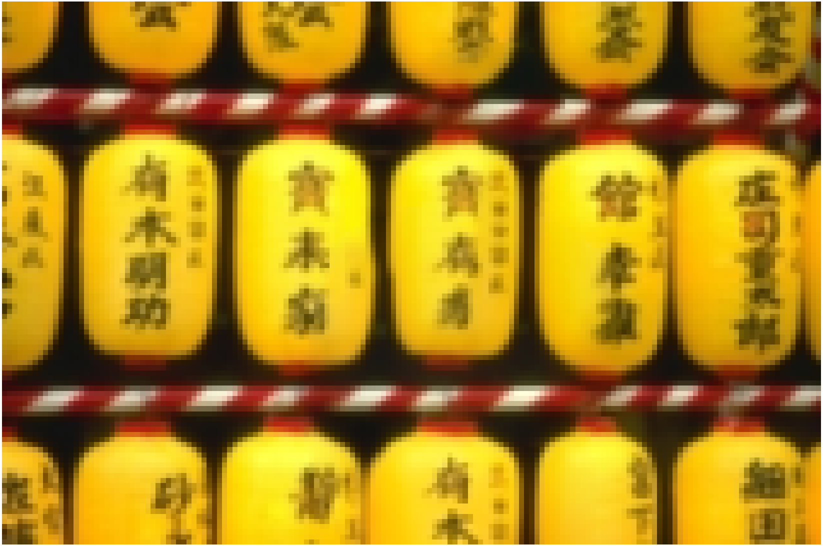

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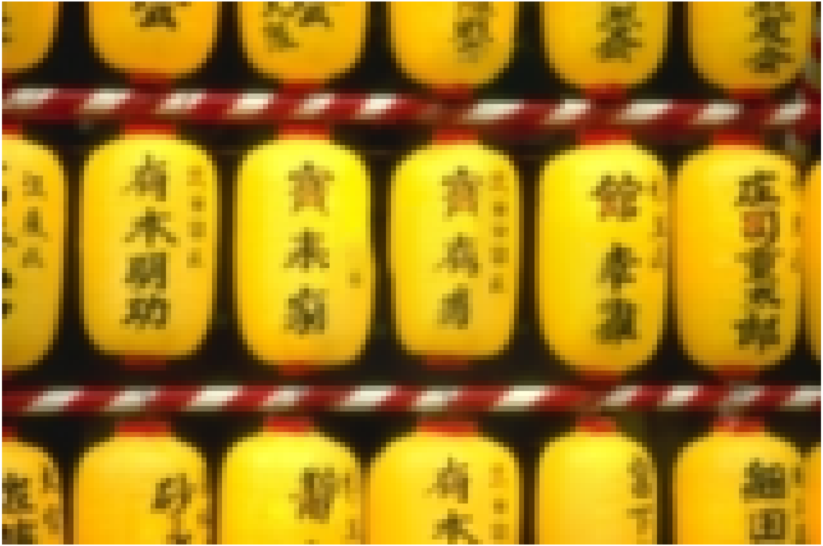

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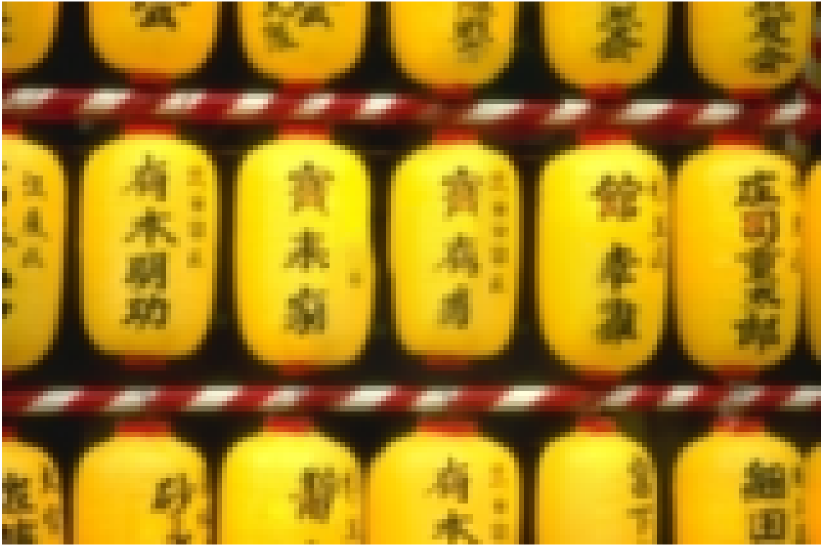

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Inpainting: Examples

A simple example in a realistic context: take out a plackard on a door.

- Original sequence.
- Inpainted sequence.

We want to take out the white placard on the right door.

Complex interaction with the persons.

Inpainting: post-production tool. Original sequence.

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Plan

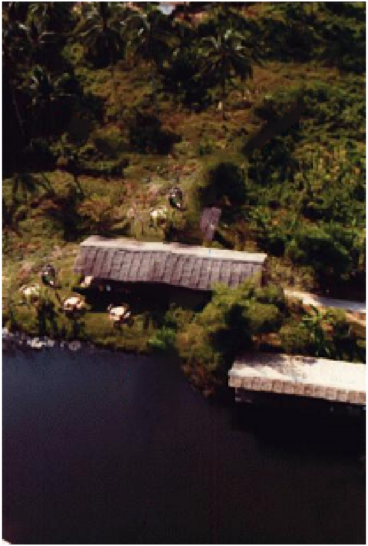
- **Problem statement**. Geometry and Texture (Exemplar-Based) Methods.
- **Basic idea: self-similarity** and its variational formulation.
- **Energies** for Exemplar-Based Inpainting: Fuzzy correspondences.
- **Energies** for Exemplar-Based Inpainting: **The Copy/Paste case.**
- The **structure** of the correspondence (copying) map.
- Algorithm. **PatchMatch:** An algorithm for fast search of patches.
- Algorithm. **Graph cut**
- **Experiments.**
- **Other Applications: Stereo Inpainting. Video.**

Inpainting: Problem statement

Definitions:

- image: $u : \Omega \to \mathbb{R}$
- image domain: Ω
- inpainting domain: O
- known data: $O^c = \Omega \setminus O$
- Patch domain: Ω_p
- Patch of u centered at x: $p_u(x)$

Inpainting: Problem statement



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Objective:

- **Visually plausible** completion
- Using only the data in O^c, or in a database

Local inpainting methods

Local Methods:

- **Continuation of level lines or gradients.** Variational methods or PDE Based (initiated by S. Masnou and J.M. Morel).

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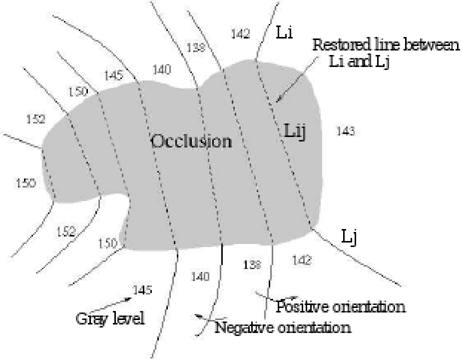
Reconstruction of the **Topographic Map** (the level lines of the image).

Dis-occlude the hidden image



- Good results on **smooth** images, fails on **textured** images.

Local inpainting methods

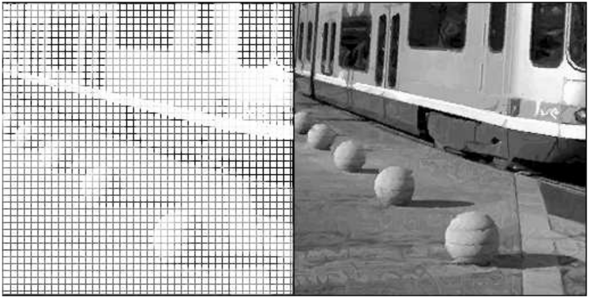


Figure: Courtesy of Simon Masnou

Non-Local or **Exemplar-Based** Methods:

- Triggered by works on **texture-synthesis** [Efros'99, Wei'00, Bonard'02].

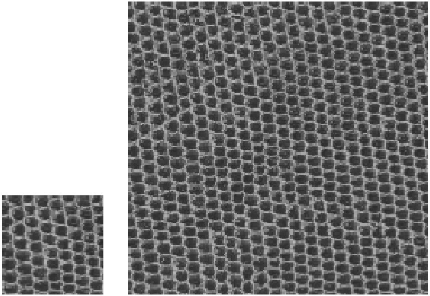


Figure: From Efros-Leung paper

Exemplar-Based Inpainting: Texture synthesis

Non-Local or **Exemplar-Based** Methods:

- Triggered by works on **texture-synthesis** [Efros'99, Wei'00, Bonard'02].
- Underlying assumption: image **self-similarity**.
- **Image Patches** as basic units of information.
- A very powerful method.
- Also in . . .
  - denoising: non-local means, UINTA [Buades-Coll-Morel 05, Awate-Withaker 06].
  - superresolution [Protter'09].
  - inspired a lot of work on variational denoising and restoration [Kinderman-Osher-Jones'06, Gilboa-Osher'06, Peyré'09].

Basic idea: self-similarity and its variational formulation.

Idea: Maximize the **self-similarity**

Correspondence map: $\varphi : O \to O^c$ [Demanet'03]

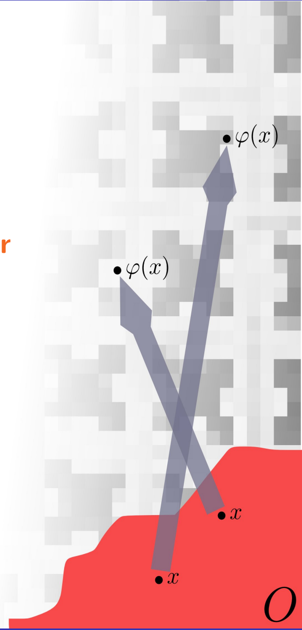
Image synthesis: $u|_O(x) = u(\varphi(x))$

Energy: **(Synthesized) Patch around x is similar to Patch around $\varphi(x)$**

$$E(\varphi) = \int_O \int_{\Omega_p} |u|_O(x+y) - u(\varphi(x)+y)|^2 \mathrm{d}y\mathrm{d}x$$

- Iteration of [Efros'99] scheme [Demanet'03]

Image and mapping joint minimization
[Wexler'07][Peyré'08][Arias'09][Kawai'09]



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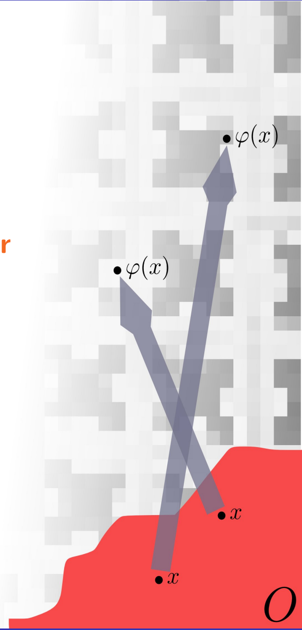


$u_0(x)$ given (noisy) image

$$u_0(x) = u(x) + n(x),$$

where $n(x)$ is a white Gaussian noise.

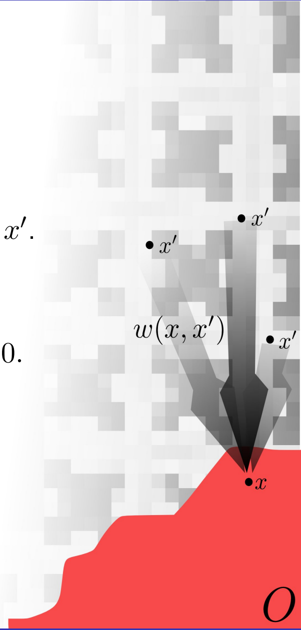
w(x,x')= weight expressing the **similarity** of x and x'.

Example

$$w(x, x') \propto \exp\left(-\frac{1}{T}\|p_{u_0}(x) - p_{u_0}(x')\|^2\right) \quad T > 0.$$

The Non-Local Means formula is [Buades-Coll-Morel'06]

$$u = NLM(u_0) := \int_\Omega w(x, x')u_0(x')\, dx'.$$



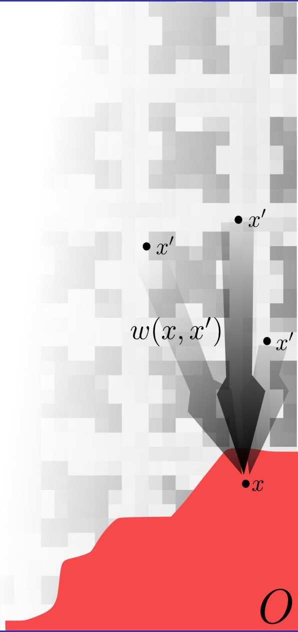
Denoising: Self-similarity and Energy for *Non-Local Means*

A non-local energy.
Let E_w the **coherence** energy [Gilboa-Osher'06].

$$E_w(u) = \int_\Omega \int_\Omega w(x, x')(u(x) - u(x'))^2 \mathrm{d}x' \mathrm{d}x$$

We are assuming that **weights are fixed**.
They penalize pixel errors:

minimizing the energy $\Longrightarrow u(x') \approx u(x)$
when $w(x, x')$ is large.



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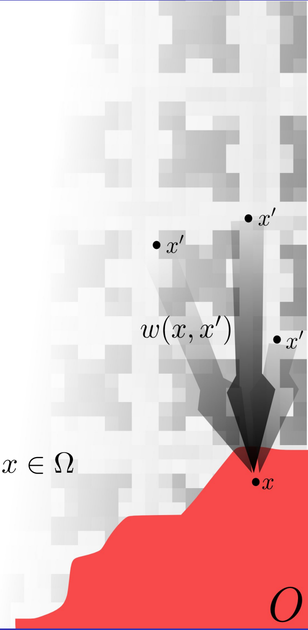
The Euler-Lagrange equation:

$$\frac{\partial E_w}{\partial u} = 0 \quad \Longleftrightarrow \quad u(x) = \int_\Omega w(x,x') u(x') \mathrm{d}x' \qquad x \in \Omega$$



A non-local energy.

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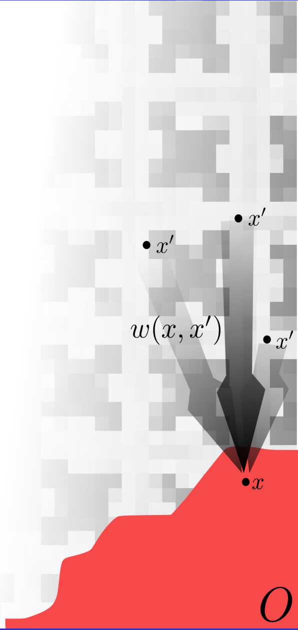
$$E_w(u) = \int_\Omega \int_\Omega w(x,x')(u(x) - u(x'))^2 \mathrm{d}x' \mathrm{d}x$$

$$u^{k+1}(x) = \int_\Omega w(x,x') u^k(x') \mathrm{d}x' \qquad x \in \Omega.$$

The iteration $k = 0$ gives NLM.



Energies for Exemplar-Based Inpainting

Energy for (w, u) (Gibbs free energy)

Consider $w : \widetilde{O} \times \widetilde{O}^c \to \mathbb{R}^+$ as a **variable**, $T \geq 0$:

$$\mathcal{E}_T(u, w) = \underbrace{\int_{\widetilde{O}} \int_{\widetilde{O}^c} w(x, x') \|p_u(x) - p_u(x')\|^2 \mathrm{d}x' \mathrm{d}x}_{\text{image energy}}$$

subject to $\displaystyle\int_{\widetilde{O}^c} w(x, x') \mathrm{d}x' = 1.$

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subject to $\displaystyle\int_{\widetilde{O}^c} w(x, x') \mathrm{d}x' = 1.$

- We compare patches
- We transfer information from O^c to O
- Weights are unknown: are computed

Energies for Exemplar-Based Inpainting

Energy for (w, u) (Gibbs free energy)

Consider $w : \widetilde{O} \times \widetilde{O}^c \to \mathbb{R}^+$ as a **variable**, $T \geq 0$:

$$\mathcal{E}_T(u, w) = \underbrace{\int_{\widetilde{O}} \int_{\widetilde{O}^c} w(x, x') \|p_u(x) - p_u(x')\|^2 \mathrm{d}x' \mathrm{d}x}_{\text{image energy}} - T \underbrace{\int_{\widetilde{O}} H(w(x, \cdot)) \mathrm{d}x}_{\text{entropy}}$$

subject to $\displaystyle\int_{\widetilde{O}^c} w(x, x') \mathrm{d}x' = 1.$

Entropy of $w(x, \cdot)$: $H(w(x, \cdot)) = -\displaystyle\int_{\widetilde{O}^c} w(x, x') \log w(x, x') \mathrm{d}x'$

$$\frac{\partial \mathcal{E}_T}{\partial w} = 0 \quad \Longleftrightarrow \quad w(x, x') = \frac{1}{q(x)} \exp\left(-\frac{1}{h}\|p_u(x) - p_u(x')\|^2\right)$$

Probabilistic correspondences may be necessary

A sparsely sampled image

Energies for Exemplar-Based Inpainting

Generalization:

$$\mathcal{E}_T(u,w) = \int_{\widetilde{O}} \int_{\widetilde{O}^c} w(x,x')U(x,x')\mathrm{d}x'\mathrm{d}x - T\mathcal{H}(w)$$

- **Patch NL-means** : $U(x,x') = \|p_u(x) - p_u(x')\|_2^2$
- **Patch NL-medians** : $U(x,x') = \|p_u(x) - p_u(x')\|_1$
- **Patch NL-Poisson** :

$$U(x,x') = \qquad \|p_{\nabla u}(x) - p_{\nabla u}(x')\|_2^2$$

- **Patch NL-Gradient Medians** :

$$U(x,x') = \qquad \|p_{\nabla u}(x) - p_{\nabla u}(x')\|_1$$

Energies for Exemplar-Based Inpainting

Generalization:

$$\mathcal{E}_T(u,w) = \int_{\widetilde{O}} \int_{\widetilde{O}^c} w(x,x')U(x,x')\mathrm{d}x'\mathrm{d}x - T\mathcal{H}(w)$$

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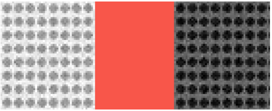
- **Patch NL-medians** : $U(x,x') = \|p_u(x) - p_u(x')\|_1$
- **Patch NL-Poisson** : $\lambda \in [0,1]$

$$U(x,x') = (1-\lambda)\|p_{\nabla u}(x) - p_{\nabla u}(x')\|_2^2 + \lambda\|p_u(x) - p_u(x')\|_2^2$$

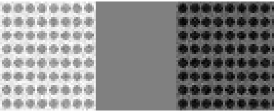
- **Patch NL-Gradient Medians** : $\lambda \in [0,1]$

$$U(x,x') = (1-\lambda)\|p_{\nabla u}(x) - p_{\nabla u}(x')\|_1 + \lambda\|p_u(x) - p_u(x')\|_2^2$$

Summary: four inpainting schemes

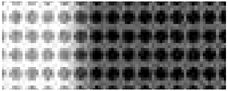


inpainting problem

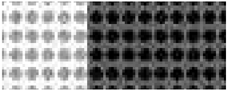


initialization

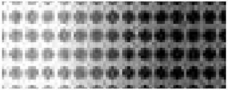
Patch NL-means (L^2 comparison)



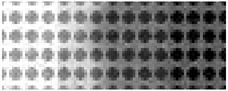
Patch NL-medians (L^1 comparison)



Patch NL-Poisson (L^2 grad comparison)



Patch NL-GM (L^1 grad comparison)



Existence of minima:

For Patch-NLM:

$$U(x,x') = \|p_u(x) - p_u(x')\|^2 = g * (u(x+\cdot) - u(x'+\cdot))^2.$$

For Patch NL-Poisson:

$$U(x,x') = \|p_{\nabla u}(x) - p_{\nabla u}(x')\|^2 = g * (\nabla u(x+\cdot) - \nabla u(x'+\cdot))^2.$$

Proposition

If $\nabla g \in L^1$ and $u \in BV(O^c)$, there are minima (u,w) of the Patch-NLM energy. For any minimum: $u \in W^{1,\infty}(O)$, $w \in W^{1,\infty}(\widetilde{O} \times \widetilde{O}^c)$.

For the NL-Poisson energy: If $\nabla g \in L^\infty$ and $u \in W^{2,2}(O^c)$, then $u \in W^{1,p}(O)$ for any $p < \infty$, $w \in W^{1,\infty}(\widetilde{O} \times \widetilde{O}^c)$.

Similar results hold when using L^1 norm.

Limit case $T \to 0$

As $T \to 0$, the energy $\mathcal{E}_T$ Gamma converges to

$$\mathcal{E}_0(u, \nu) := \int_{\widetilde{O}} \int_{\widetilde{O}^c} U(x, x') d\nu(x, x'),$$

where $x \to \nu(x, \cdot)$ is a measurable probability-valued map (indeed a Young measure).

Proposition There are minima (u, ν) of $\mathcal{E}_0$.
There are minima (u, ν) such that ν is given by a correspondence map, i.e. there is a measurable map $\varphi : \widetilde{O} \to \widetilde{O}^c$ such that

$$\varphi(x) \in \arg\min_{x' \in \widetilde{O}^c} U(x, x'),$$

$$\nu(x, x') = \nu^{\varphi}(x, x') = \delta(x' - \varphi(x)).$$

Limit case $T \to 0$: Existence

Correspondence case: The Proposition is a consequence of Kuratowski-Ryll-Nardewski Theorem.

Or: since the extremal points of the set of Young measures are the $\{\nu^\varphi : \varphi\}$.

The correspondence case (relaxation)

For correspondences the energy can be written as

$$\mathcal{E}_0(u,\varphi) = \int_O \int_{\Omega_p} |u|_O(x+y) - u(\varphi(x)+y)|^2 \mathrm{d}y\mathrm{d}x.$$

The two unknowns are $u|_O$ and φ and one can proceed by iterated optimization.

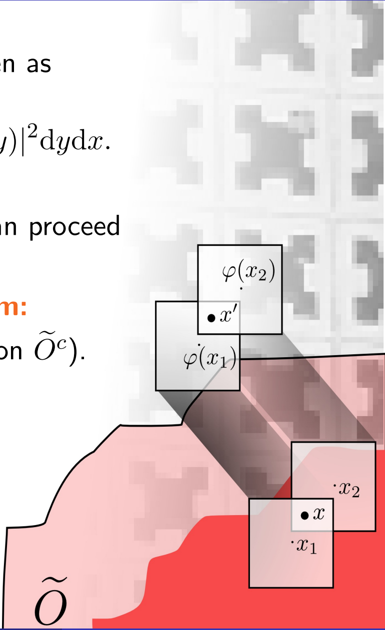
Algorithm: Iterated optimization algorithm:

Initialization: choose u^0 as the average of u on $\widetilde{O}^c$).

For each $k \in \mathbb{N}$ solve

$$\nu^k = \arg\min_\nu \mathcal{E}_0(u^k,\nu),$$

$$u^{k+1} = \arg\min_u \mathcal{E}_0(u,\nu^k),$$



The correspondence case (relaxation)

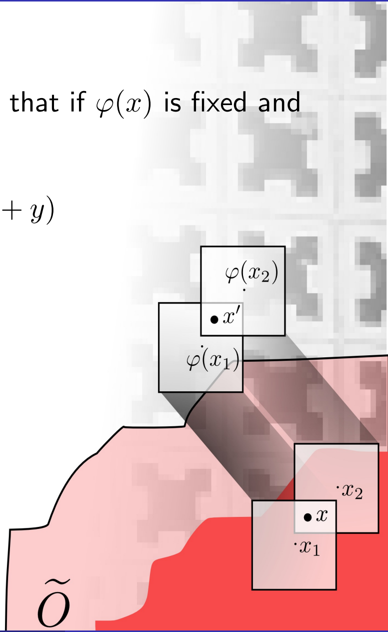
For the case of patch NL-means: We observe that if $\varphi(x)$ is fixed and $u|_O = \arg\min_u \mathcal{E}_0(u, \varphi)$, then

$$u|_O(x) = \sum_{\Omega_p} g(y) u(\varphi(x-y)+y)$$

When φ is a translation, *i.e.* $\varphi(x) = x + t$:

$$u|_O(x) = u(\varphi(x))$$

Analogous for NL-Poisson case.



Correspondence case: Structure of the solution

Experimental evidence for Piece-wise translation φ

$\varphi(x) = x + t(x)$, with t piece-wise constant.

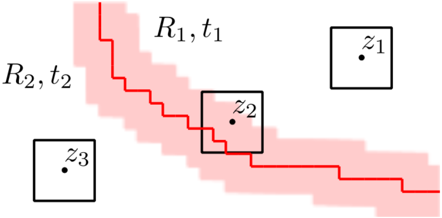
Correspondence case: Structure of the solution

Experimental evidence for Piece-wise translation φ

$\varphi(x) = x + t(x)$, with t piece-wise constant.

Copy regions and transition band.

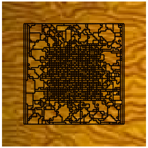
Regions R_1 and R_2 have a constant t. Data is rigidly translated (copied) from corresponding regions in O^c. Red band: transition between copy regions.



L^1: Sharp transitions (between intensity/gradients)

L^2: Smooth blending by averaging (of intensity/gradients)

Emergence of copy regions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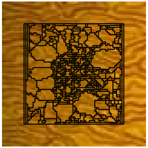
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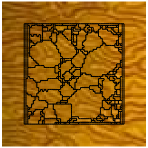
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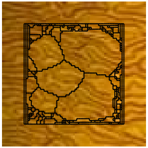
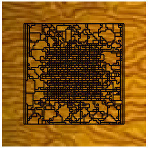
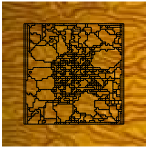
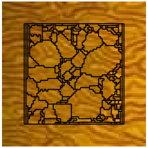
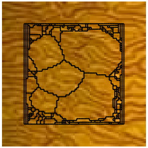
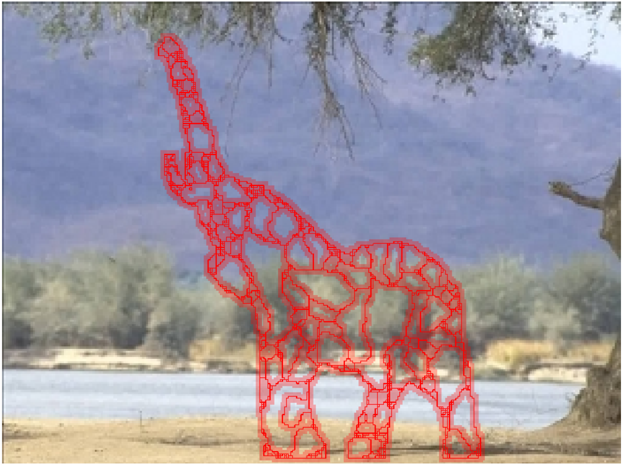

34

Emergence of copy regions thoughout the iterations

Emergence of copy regions

 1  2  3···  34

Emergence of copy regions thoughout the iterations



Empirical observation:
φ often results in a piece-wise traslation.

Result obtained with a multiscale scheme.

Existence of Mildly Regular Correspondences

Theorem (Extension of KRN)

- $X \subset \mathbb{R}^N$ open bounded with Lipschitz boundary, $Y \subset \mathbb{R}^m$ compact.
- $U : X \times Y \to \mathbb{R}$ be a Lipschitz continuous function. Let

$$x \in X \to M(x) := \{y \in Y : U(x,y) = \min_{\bar{y} \in Y} U(x, \bar{y})\}.$$

Then there exists a selection of the multifunction $x \in X \to M(x) \subseteq Y$, i.e., a function $\varphi : X \to Y$ such that $\varphi(x) \in M(x)$ for all $x \in X$, which is a uniform limit of functions with finitely many values in $BV(X)^m$.

Thus $\mathcal{H}^{N-1}$-a.e. $x \in X$ is either a point of approximate continuity of φ, or a jump point with two (lateral limits) limits. Its jump set J_φ is a countably rectifiable set.

Example: Emergence of copy regions

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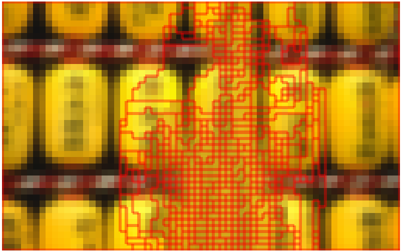

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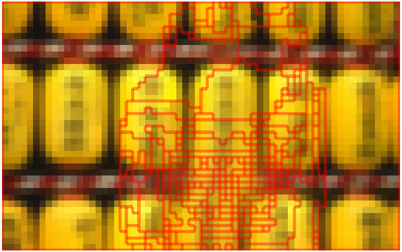

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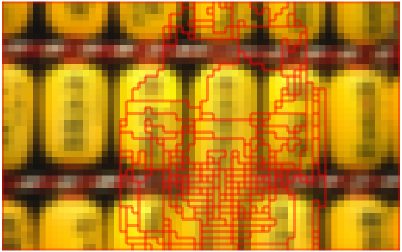

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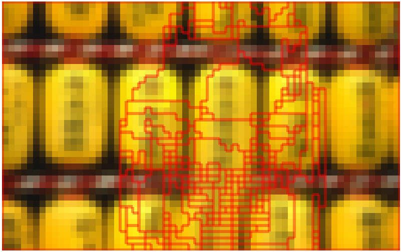

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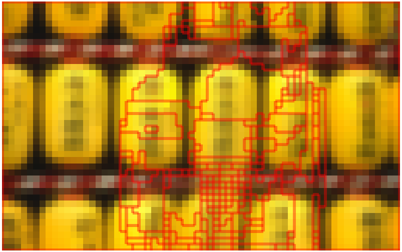

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 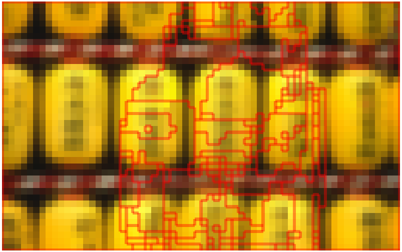

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 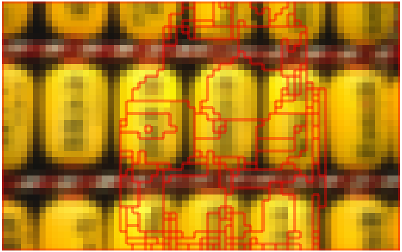

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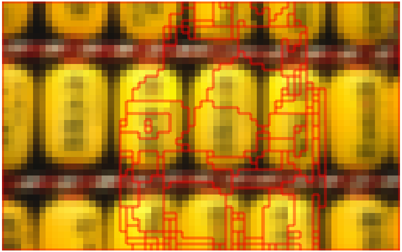

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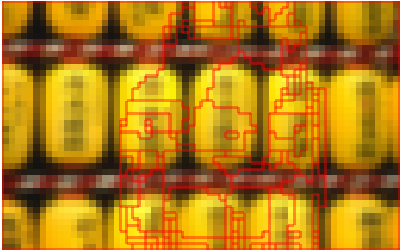

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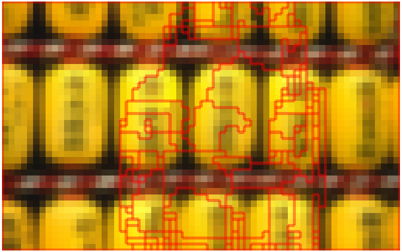

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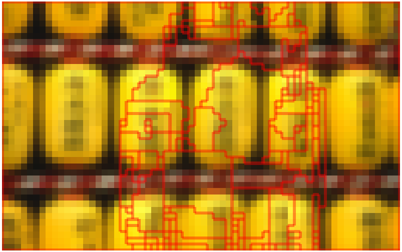

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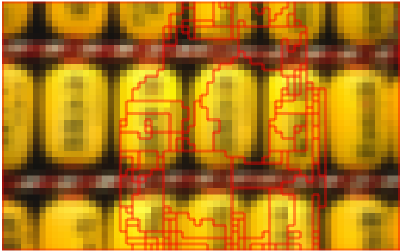

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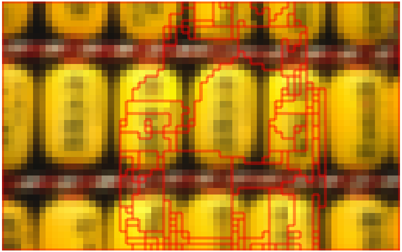

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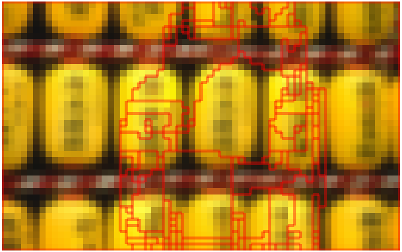

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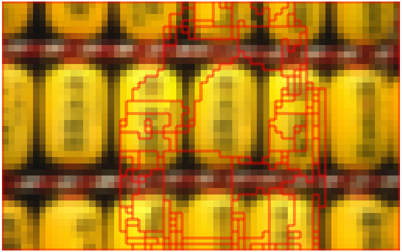

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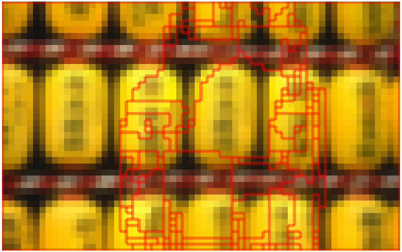

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 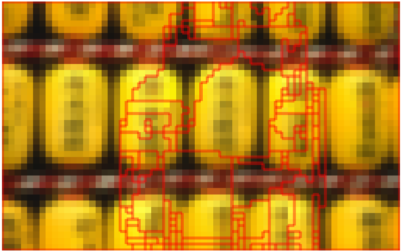

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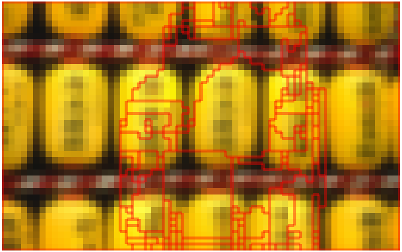

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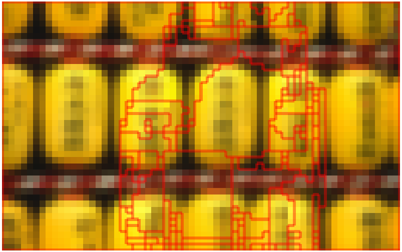

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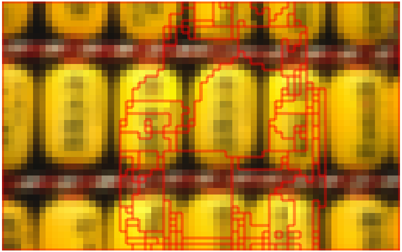
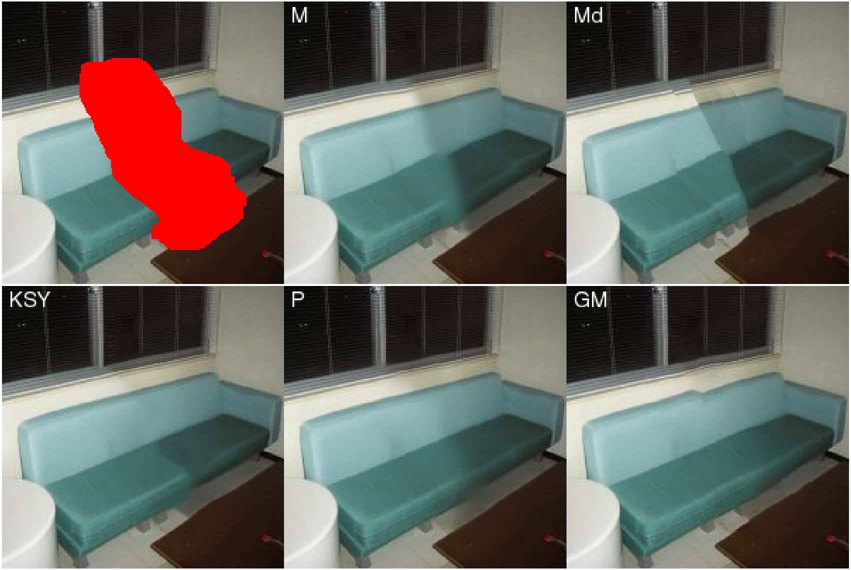


Experiments



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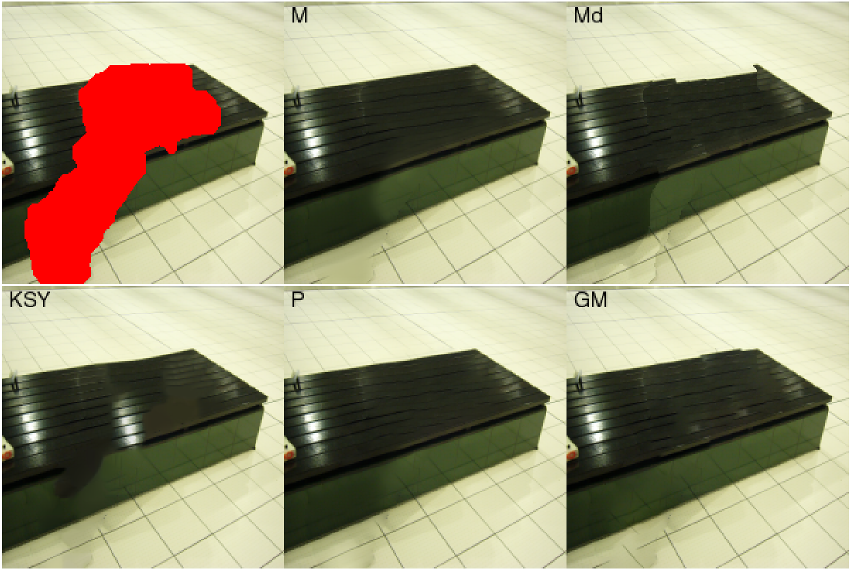

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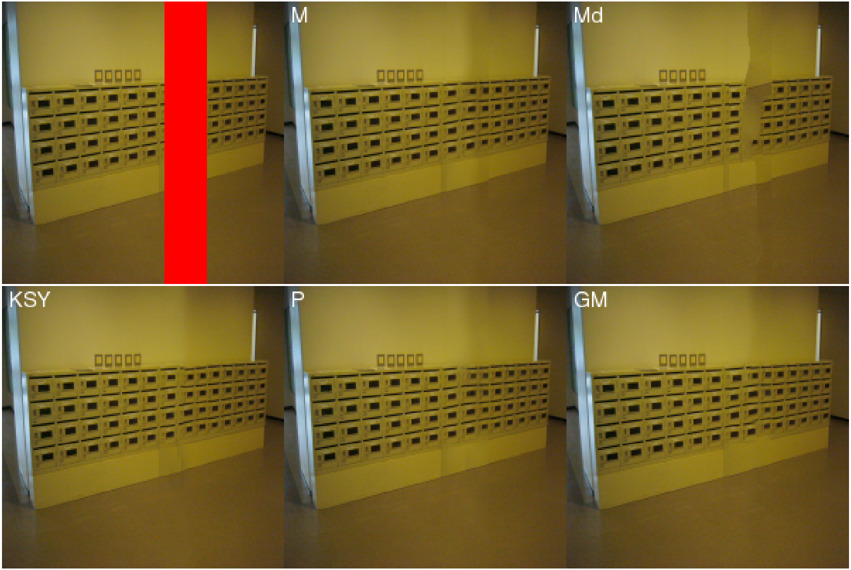


Experiments

Results with non-variational scheme

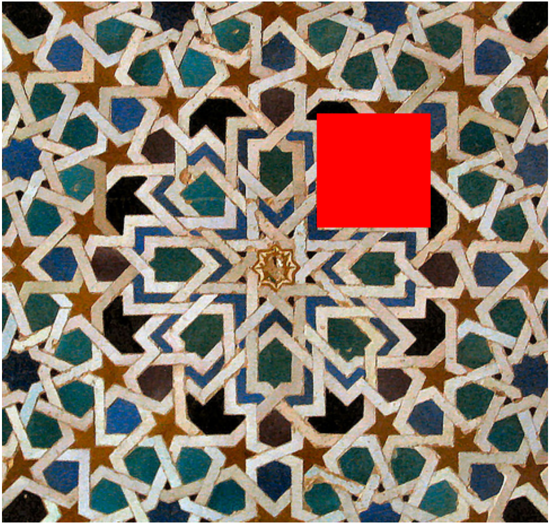
Departure from variational model:

1. Synthesize image using $\lambda_u << 1$.
2. Compute weights using λ_w. Parameter to be set for each image.

Greater flexibility, but it is not variational.

An example with symmetries

We generate symmetric versions of the image



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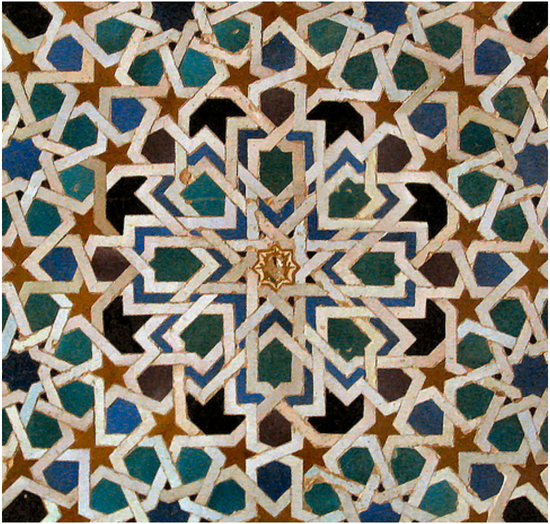

An example of reconstruction of a sparsely sampled image

Reconstruction of a sparsely sampled image (Annealing)

An example of reconstruction of a sparsely sampled image

Reconstruction of a sparsely sampled image (Annealing)

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Reconstruction of a sparsely sampled image (Annealing)

A different algorithmic perspective: using graph cuts

Reformulation of Efros-Leung-Demanet-Chan energy.

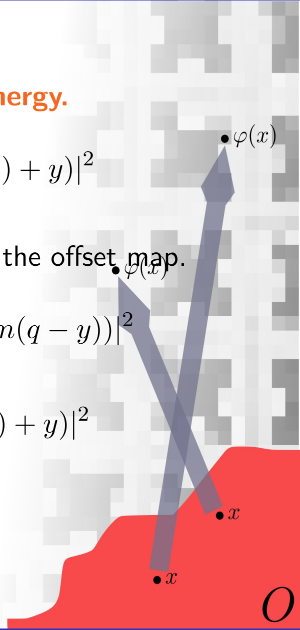
$$E(\varphi) = \sum_{O} \sum_{\Omega_p} |u(\varphi(x+y)) - u(\varphi(x)+y)|^2$$

Let $\varphi(x) = x + m(x)$ where $m : \Omega \to \mathbb{Z}^2$ represents the offset map.

$$E(m) = \sum_{q \in O} \sum_{y \in N_p} |u(q+m(q)) - u(q+m(q-y))|^2$$

$$+ \sum_{q \in \partial O} \sum_{y \in \Omega_p} |u(q+y) - u(q+m(q)+y)|^2$$

Minimization using graph cuts. (Y. Liu, V.C.)



Inpainting using Efros model based on pixels

Using Efros model based on pixels

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Using Efros model based on pixels

Inpainting using Efros model based on pixels

Using Efros model based on pixels

Using Efros model based on pixels

Figure: Using Komodakis-Tziritas

Using Efros model based on pixels

Inpainting using Efros model based on pixels

Using Efros model based on pixels

Using Efros model based on pixels

Using Efros model based on pixels

Problems

- Further analysis of the “copy regions”
  - There are better regularity properties for φ?
  - Should we enforce them? (as in [Demanet’03,Aujol’08])
  - Can we prove that the algorithm produce piecewise regular solutions ?

Stereo Image Inpainting

Given a pair of stereo images (or videos) : we want to eliminate an object in both images of the pair with

- **Depth consistency in both images** (good depth perception)
- Reconstruction of the depth of dis-occluded objects.

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Stereo Image Inpainting: another example

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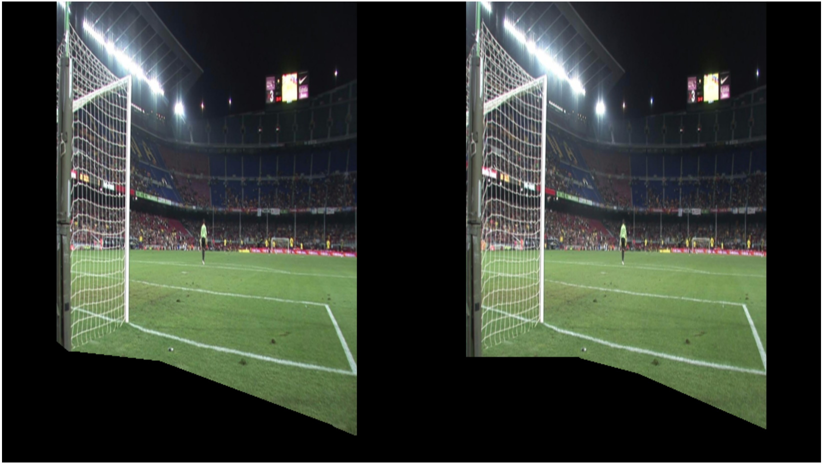


Stereo Image Inpainting: another example

Stereo Image Inpainting

Ingredients:

- Depth computation (assuming cameras are calibrated)
- Consistent selection of the inpainting regions
- **Depth inpainting**
- **Simultaneous inpainting** of both images: incorporate depth consistency into the inpainting energy

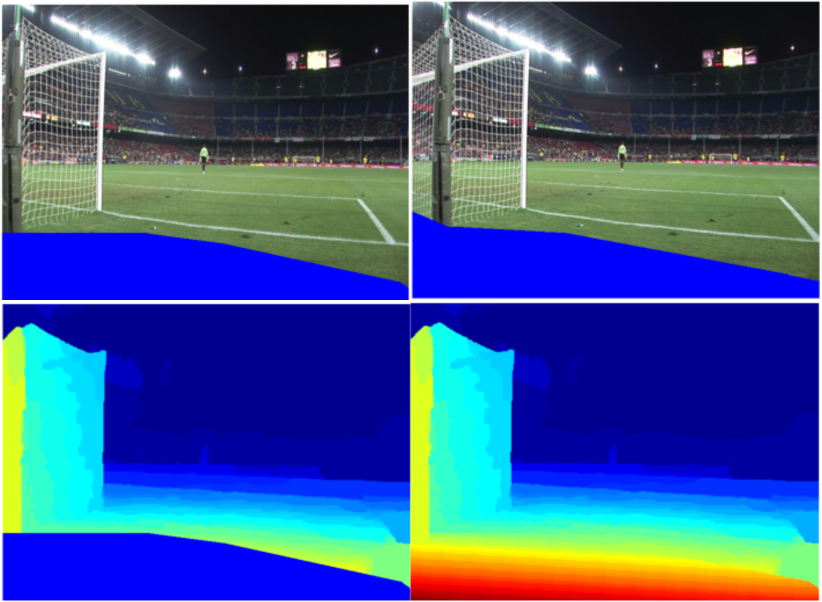
Stereo Image Inpainting

Ingredients:

- Depth computation (assuming cameras are calibrated)
- Consistent selection of the inpainting regions
- **Depth inpainting**: region models
- **Simultaneous inpainting** of both images I_1, I_2: incorporate depth consistency into the inpainting energy

$$\sum_{x \in O} \|p_{I_1}(x) - p_{I_1}(\varphi(x))\|_{g,2}^2 + \|p_{I_2}(x + d(x)) - p_{I_2}(\varphi(x) + d(\varphi(x)))\|_{g,2}^2.$$

Stereo Image Inpainting



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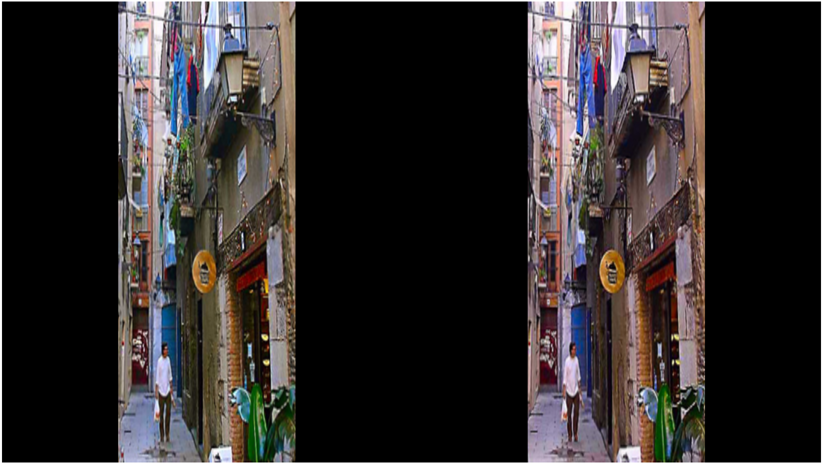

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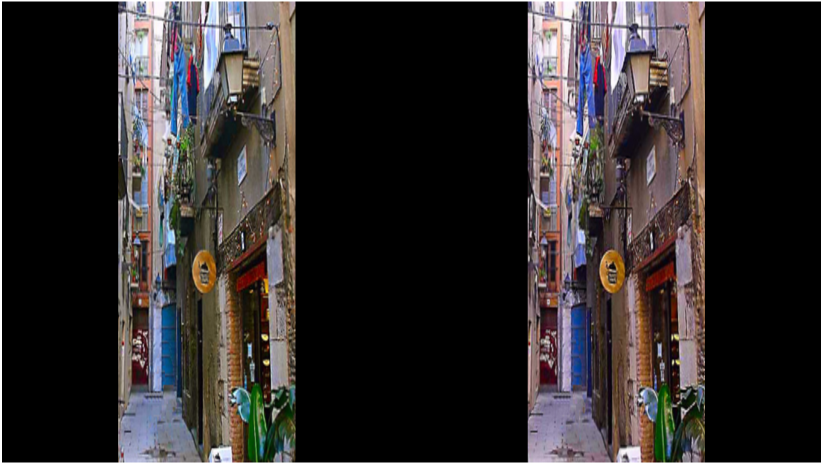

Stereo Image Inpainting

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Stereo Image Inpainting

Object insertion in video sequences

Let v be the optical flow. **We insert an object in a video:** in O (a space-time domain). We forward it by minimizing

$$E_\lambda(u) = \int_O \left(\frac{1}{2} \|\nabla_x \partial_v u(x,t)\|^2 + \frac{\lambda}{p} \|\nabla_x u(x,t)\|^p \right) \mathrm{d}x \, \mathrm{d}t,$$

with $\lambda \geq 0$, $p = 1, 2$.

We denoted $\partial_v = \partial_t + v \nabla_x$.

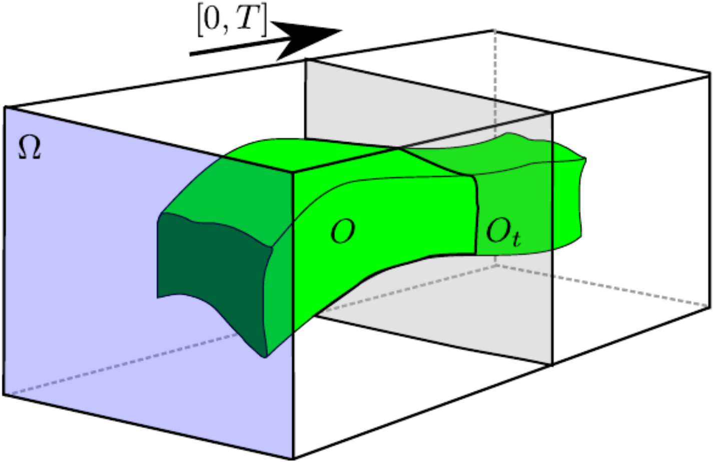
We assume (as in DiPerna-Lions)

$$v \in L^1(0,T; W^{1,1}_{\mathrm{loc}}(\mathbb{R}^2;\mathbb{R}^2)) \cap L^1(0,T; L^\infty(\mathbb{R}^2;\mathbb{R}^2)), \tag{1}$$

$$\operatorname{div} v \in L^1(0,T; L^\infty(\mathbb{R}^2)). \tag{2}$$

(For existence if $\lambda = 0$, for uniqueness).

Object insertion in video sequences



Object insertion in video sequences

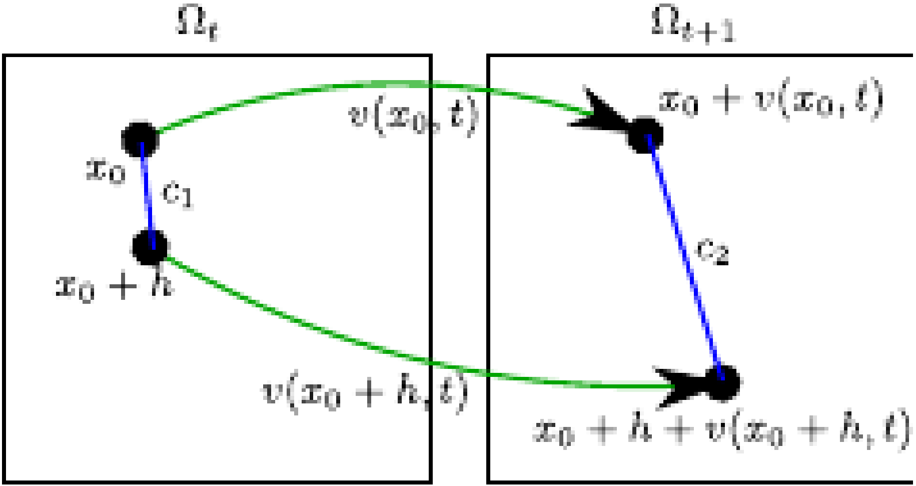
Explanation:

$$\nabla_x \partial_v u(x,t) = 0\,. \tag{3}$$

Our variational model is based on this equation. A Taylor expansion of (3) leads to

$$\begin{aligned} u(y_0 + kv(y_0,t), t+k) - u(x_0 + kv(x_0,t), t+k) \\ \approx u(y_0,t) - u(x_0,t)\,, \end{aligned} \tag{4}$$

Object insertion in video sequences



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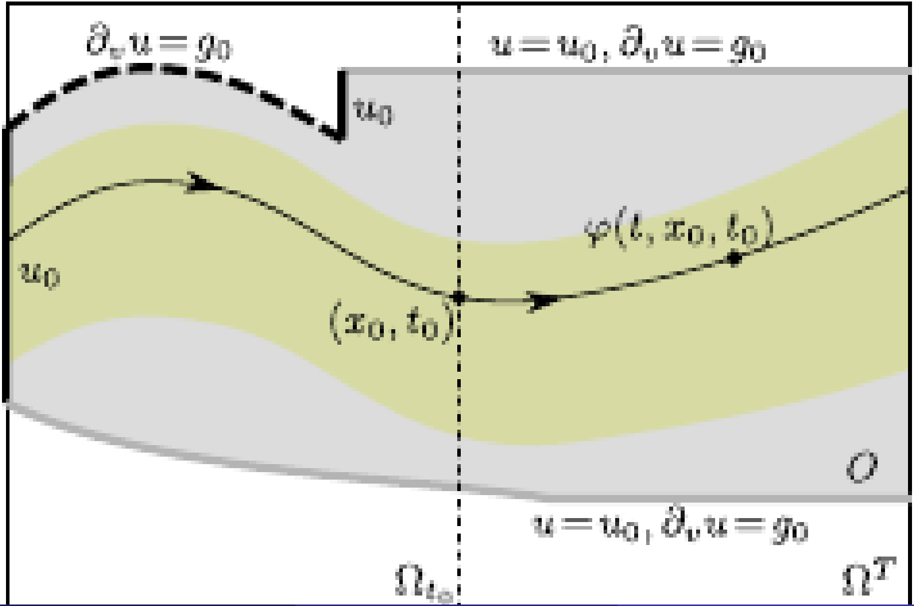

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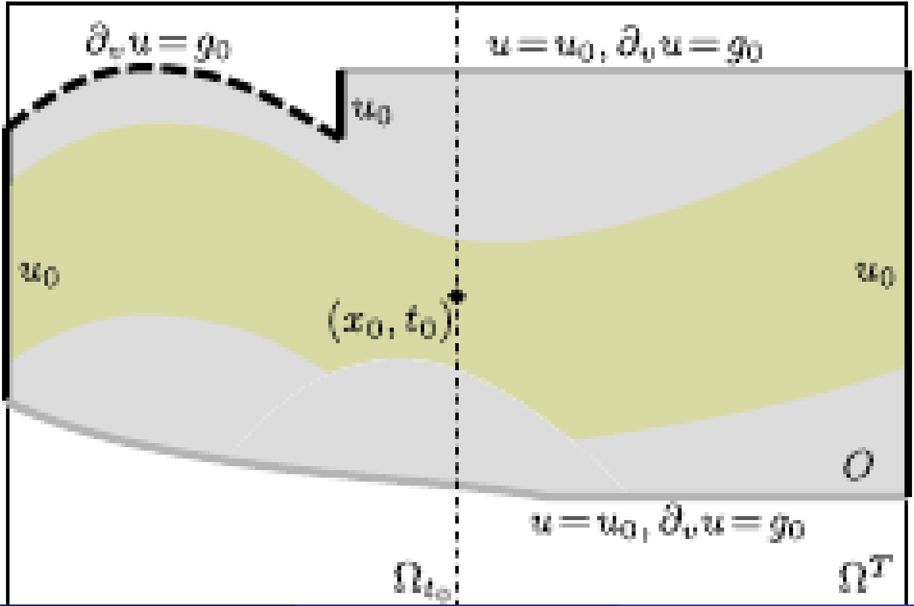

Object insertion in video sequences

Boundary conditions for the one-lid setting.
If we insert an object in the first frame: we choose the set of boundary conditions

$$u(x,0) = u_0(x,0), \qquad x \in O_0, \tag{5}$$

$$u(x,t) = u_0(x,t), \qquad (x,t) \in \partial O_{\mathsf{vert}}, \tag{6}$$

$$\partial_v u(x,t) = g_0(x,t) \;, \qquad (x,t) \in \partial O_{\mathsf{tang}} \setminus \partial\Omega^T, \tag{7}$$

$$\begin{matrix} u(x,t) = u_0(x,t) \\ \partial_v u(x,t) = g_0(x,t) \end{matrix} \;, \qquad (x,t) \in \partial O_{\mathsf{obli}} \setminus \partial\Omega^T, \tag{8}$$

Object insertion in video sequences

The boundary conditions on the rest of ∂O are

$$\nabla_x^*(\kappa \nabla_x \partial_v u)(x,t) = 0, \qquad x \in O_T, \tag{9}$$

$$\begin{array}{l} \lambda\xi \cdot \nu^{O_t}(x,t) = 0 \\ \kappa \nabla_x \partial_v u(x,t) \cdot \nu^{O_t}(x,t) = 0 \end{array}, \qquad (x,t) \in \partial O_{\mathsf{tang}} \cap \partial\Omega^T, \tag{10}$$

$$\begin{array}{l} \nabla_x^*(\kappa \nabla_x \partial_v u)(x,t) + \lambda\xi \cdot \nu^{O_t}(x,t) = 0 \\ \kappa \nabla_x \partial_v u(x,t) \cdot \nu^{O_t}(x,t) = 0 \end{array}, \quad (x,t) \in \partial O_{\mathsf{obli}} \cap \partial\Omega^T. \tag{11}$$

$\kappa = 1$

Boundary conditions for the two-lid setting.
They are given by (5),(6),(7),(8),(10),(11), and (9) is replaced by

$$u(x,T) = u_0(x,T) \quad \text{in } O_T. \tag{12}$$

Thank you