Ergodic Properties of Square-Free Numbers

Abstract: We study binary and multiple correlations for the set of square-free numbers and we construct a dynamical systems naturally associated to them. We prove that such dynamical system has pure point spectrum and it is therefore isomorphic to a translation on a compact abelian group. In particular, the system is ergodic but not weakly mixing, and it has zero metric entropy. The latter results were announced recently by Peter Sarnak and our strategy provides an alternative approach (joint work with Yakov Sinai). One can then generalize this to number fields and realize the set of square-free integers as a typical realization of a Z^d action with zero entropy and pure point spectrum (joint work with Ilya Vinogradov).