Computational Dynamics and Computer Assisted Proofs

Organizers: Warwick Tucker (*Uppsala University, SE)* Piotr Zgliczyński (*Jagiellonian University)*

Tuesday, July 3, 15:45–17:45, Medium Hall A

Talks:
Alain Albouy (*CNRS, Observatoire de Paris, FR*), **Rigorous upper bounds for the number of equilibrium configurations of n point particles**

Angel Jorba (*Universidad de Barcelona, ES*), **Approximating invariant tori on a parallel computer**

Michael Plum (*Karlsruhe Institute of Technology, DE*), Coauthors: Kaori Nagatou, P. Joseph McKenna, **Orbital stability investigations for travelling waves in a nonlinearly supported beam**

Daniel Wilczak (*Jagiellonian University, Kraków, PL*), **Computing of families of invariant tori surrounding elliptic periodic orbits**

Rigorous upper bounds for the number of equilibrium configurations of n point particles

Alain Albouy
CNRS, Observatoire de Paris, FR

Several physical situations lead to the consideration of n interacting points, constrained or not. Giving masses, charges, vorticities, etc., and looking for the equilibria may lead to a long list of configurations with impredictible patterns. The oldest example of such a list could be Mayer's study of configurations of "floating magnets" in 1878. A year earlier E.J. Routh published the first results on the relative equilibria of four celestial bodies interacting according to Newton's law. In 2005, Hampton and Moeckel proved that such relative equilibria are finitely many, whatever be the masses, thus answering the case $n = 4$ of Smale's 6th mathematical problem for the 21st century.

Their proof used several programs (Mathematica, Porta, Macaulay2, lrs, Mixvol). In a recent work with Vadim Kaloshin, we re-proved their result without computer. But, while passing to $n = 5$, part of our proof appeared to need handling of big polynomials with integer coefficients. Actually we gave the finiteness result except on a codimension 2 algebraic subset of the mass space with a complicated equation.

Approximating invariant tori on a parallel computer

Angel Jorba
Universidad de Barcelona, ES

The numerical approximation of invariant tori of flows (or maps) is a computationally intensive task, specially when the tori is of dimension strictly larger than 2 (or 1 for maps). In this talk we will discuss several ways of taking advantage of the parallel capacities offered by multicore computers and also by clusters. We will use examples coming from problems of Astrodynamics, that require the computation of 3-D and 4-D invariant tori.

Orbital stability investigations for travelling waves in a nonlinearly supported beam

Michael Plum
Karlsruhe Institute of Technology, DE

We consider the fourth-order problem

$$\varphi_{tt} + \varphi_{xxxx} + f(\varphi) = 0, \quad (x,t) \in \mathbf{R} \times \mathbf{R}^+, \tag{1}$$

with a nonlinearity f vanishing at 0. Solitary waves $\varphi = u(x+ct)$ satisfy the ODE

$$u'''' + c^2 u'' + f(u) = 0 \quad \text{on } \mathbf{R}, \tag{2}$$

and for the case $f(u) = e^u - 1$, the existence of at least 36 travelling waves was proved in [Breuer, Horák, McKenna, Plum, Journal of Differential Equations 224 (2006)] by computer assisted means. We investigate the orbital stability of these solutions via computation of their Morse indicies and using results from [Grillakis, Shatah, Strauss, Journal of Functional Analysis 74 (1987)]. In order to achieve it we make use of both analytical and computer-assisted techniques

Coauthors: Kaori Nagatou, P. Joseph McKenna

Computing of families of invariant tori surrounding elliptic periodic orbits

Daniel Wilczak
Jagiellonian University, Kraków, PL

The well-known theorem of Siegel and Moser says that a non-resonant elliptic fixed point of an analytic symplectic planar map is generically surrounded by invariant curves and thus it is stable. We will show how this theorem may be applied to verify stability of continuous families of periodic solutions for ODEs that are far from equilibria. An application to some textbooks examples such as the Henon-Heiles hamiltonian or the Michelson will be given.