Continuous Real Rational Functions and Related Topics

ORGANIZER: Krzysztof Kurdyka (*Université de Savoie, FR)*

Monday, July 2, 17:15–19:15, Medium Hall A

TALKS:

Krzysztof Kurdyka (*Université de Savoie, FR*), **Arc-analytic and continuous rational functions**

Wojciech Kucharz (*Jagiellonian University, Kraków, PL*), **Rational maps in real algebraic geometry**

Jean-Philippe Monnier (*Université d'Angers, FR*), COAUTHORS: G. Fichou, J. Huisman, F. Mangolte, **Reguluous functions**

Frederic Mangolte (*Université d'Angers, FR*), COAUTHOR: Janos Kollàr, **Approximating curves on real rational surfaces**

Arc-analytic and continuous rational functions

Krzysztof Kurdyka
Université de Savoie, FR

I will describe a background on arc-analytic semi-algebraic functions (K. Kurdyka, Math. Ann. 88) and applications to the surjectivity of injectiv real algebraic morphisms (K. Kurdyka, Injective endomorphisms of real algebraic sets are surjective Math.Ann. 313, 69Ð82, 1999,K. Kurdyka, A. Parusinski, Arc-symmetric sets and arc-analytic mappings Arc spaces and additive invariants in real algebraic and analytic geometry, 33Ð67, Panor.SynthŔses 24, Soc. Math. France, Paris, 2007.) . Finally I will present main results of a preprint of Janos KollĞr, Continuous rational functions, arXiv:1101.3737v1 [math.AG], 2010. In fact his preprint was the main inspiration for this proposal.

Rational maps in real algebraic geometry

Wojciech Kucharz
Jagiellonian University, Kraków, PL

The paper deals with rational maps between real algebraic sets. We are interested in therational maps which extend to continuous maps defined on the entire source space. In particular,we prove that every continuous map between unit spheres is homotopic to a rational map of such atype. Moreover, each continuous map from a nonsingular compact algebraic set to the sphere of the same dimension is homotopic to a continuous rational map. This result demonstrates that continuous rational maps are more flexible than regular maps. We also establish connections with algebraic cycles and vector bundles.

Reguluous functions

Jean-Philippe Monnier
Université d'Angers, FR

We study the ring of continuous rationals functions which can be continuously extended to R^n. This ring is not noetherian (as in the case of the ring of arc-analytic semi-algebraic functions). We prove a strong Nullstellensatz. We give also a characterization of the prime ideals in this ring.

Coauthors: G. Fichou, J. Huisman, F. Mangolte

Approximating curves on real rational surfaces

Frederic Mangolte
Université d'Angers, FR

It is known that every differentiable map from the circle to a rational variety $S^1 \to X$ can be approximated by an algebraic map $P^1(R) \to X$. In particular, any simple closed curve on a rational surface S can be approximated by a rational curve on S. Note that the usual result is about maps of rational curves, so the image may have some extra isolated points. As a consequence of our theorem of approximation of diffeomorphisms by algebraic automorphisms, we get rid of these.Recall that by Comessatti's theorem, a real rational surface S is diffeomorphic to the sphere, the torus or to any nonorientable surface.Main theorem:Let R be a closed topological surface and K be a simple closed curve on R. Assume that R is either nonorientable or of genus < 2. Then the embedding $K \to R$ is diffeomorphic to an embedding $L \to S$ of a smooth rational curve L on a smooth rational surface S . Furthermore, if R is already a smooth rational surface, then K can be approximated in the C^∞-topology by a smooth rational curve. Furthermore, we give necessary and sufficient topological conditions for a simple closed curve on a rational surface S to be approximated by a (-1)-curve, or by a fiber of a conic bundle. Note that (-1)-curves are quite rigid objects, hence approximating by (-1)-curves is a subtle problem.

Coauthors: Janos Kollàr