On locally soluble AFM–groups

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Abstract

Let A be a vector space over a field F. The group $GL(F,A)$ of all automorphisms of A and its subgroups are called linear groups. If A has finite dimension over F then every element of $GL(F,A)$ defines a non–singular $(n\times n)$–matrix over F, where $n = dim_F A$. The theory of finite dimensional linear groups is one of the best developed in Algebra. In the case when $dim_F A$ is infinite, the situation is rather different. Study of infinite dimensional linear groups requires some additional restrictions. One from these restrictions is a finitarity of linear groups. We recall that a linear group G is called a finitary if for each element $g \in G$ the subspace $C_A(g)$ has finite codimension [1–2]. The theory of finitary linear groups has been developed very intensivly and have became very rich with many interesting results [2].

In [3] the authors have considered antifinitary linear groups. Let $G \leq GL(F,A)$, $A(wFG)$ be an augmentation ideal of a group ring FG. The augmentation dimension of G is the F–dimension of $A(wFG)$. A linear group G is an antifinitary linear group if each proper subgroup of G having infinite augmentation dimension is finitely generated.

If $G \leq GL(F,A)$ then A can be considered as an FG–module. The generalization of this case is consideration of $\mathbf{R}G$–module where $\mathbf{R}$ is a ring. B.A.F. Wehrfritz has considered an artinian–finitary group of automorphisms of a module M over a ring $\mathbf{R}$ and a noetherian–finitary group of automorphisms of a module M over a ring $\mathbf{R}$ wich are the analogues of finitary linear groups [4–6]. The group of automorphisms $F_1 Aut_{\mathbf{R}} M$ of a module M over a ring $\mathbf{R}$ is called artinian–finitary if for each element $g \in F_1 Aut_{\mathbf{R}} M$ $A(g-1)$ is an artinian $\mathbf{R}$–module. The group of automorphisms $FAut_{\mathbf{R}} M$ of a module M over a ring $\mathbf{R}$ is called noetherian–finitary if for each element $g \in FAut_{\mathbf{R}} M$ $A(g-1)$ is a noetherian $\mathbf{R}$–module.

We need the definition of a cocentralizer of a subgroup H in the module A [7].

Definition 1. Let A be an $\mathbf{R}G$-module where $\mathbf{R}$ is a ring, G is a group. If $H \leq G$ then the quotient–module $A/C_A(H)$ considered as an $\mathbf{R}$–module is called a cocentralizer of a subgroup H in the module A.

The generalization of classes of artinian and noetherian modules is the class of minimax modules [8].

Definition 2. An $\mathbf{R}$–module A is called a minimax $\mathbf{R}$–module if A has finite series of submodules such that each factor is either a noetherian $\mathbf{R}$–module or an artinian $\mathbf{R}$–module.

We consider the analogy of antifinitary linear groups in the theory of modules over group rings. Let A be an $\mathbf{R}G$–module. A group G is called AFM–group if each proper subgroup $H \leq G$ such that its cocentralizer in the module A is not a minimax $\mathbf{R}$–module is finitely generated.

Let $MD(G)$ be a set of all elements $x \in G$ such that the cocentralizer of a group $\langle x \rangle$ in the module A is a minimax $\mathbf{R}$–module. $MD(G)$ is a normal subgroup of a group G.

We consider AFM–groups such that $\mathbf{R} = \mathbb{Z}$ is a ring of integers and $C_G(A) = 1$. The main results are the theorems.

Theorem 1. Let A be a $\mathbb{Z}G$–module. If the cocentralizer of a group G in the module A is a minimax $\mathbb{Z}$–module then G is soluble.

Theorem 2. Let A be a $\mathbb{Z}G$–module, G is a locally soluble AFM–group. Then a group G is hyperabelian.

Theorem 3. Let A be a $\mathbb{Z}G$–module, G is a finitely generated soluble AFM–group. If the cocentralizer of a group G in the module A is not a minimax $\mathbb{Z}$–module then the following conditions hold:

(1) the cocentralizer of a subgroup $MD(G)$ in the module A is a minimax $\mathbb{Z}$–module;

(2) a group G has a normal subgroup U such that the quotient group G/U is polycyclic.

BIBLIOGRAPHY

[1] *Phillips R.E.* The structure of groups of finitary transformations. J. Algebra., **119**, no. 2 (1988), 400–448.

[2] *Phillips R.E.* Finitary linear groups: a survey. "Finite and locally finite groups". NATO ASI ser. C Math. Phys. Sci., Kluver Acad. Publ., Dordreht, **471** (1995), 111–146.

[3] *Kurdachenko L.A., Muñoz-Escolano J.M., Otal J.* Antifinitary linear groups. Forum Math., **20**, no. 1 (2008), 27–44.

[4] *Wehrfritz B.A.F.* Artinian-finitary groups over commutative rings. Illinois J. Math., **47**, no. 1–2 (2003), 551–565.

[5] *Wehrfritz B.A.F.* Artinian-finitary groups are locally normal-finitary. J. Algebra, **287**, no. 2 (2005), 417–431.

[6] *Wehrfritz B.A.F.* Artinian-finitary groups over commutative rings and non-commutative rings. J. Lond. Math. Soc.(2), **70**, no. 2 (2004), 325–340.

[7] *Kurdachenko L.A.* On groups with minimax classes of conjugate elements. Infinite groups and adjoined algebraic systems. Kyev: National Academy of Sciences of Ukraine, Institute of Mathematics. – 1993. – 160–177.

[8] *Kurdachenko L.A., Subbotin I.Ya., Semko N.N.* Insight into Modules over Dedekind Domains. Kyev: National Academy of Sciences of Ukraine, Institute of Mathematics. – 2008. – 119 p.

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