Differential Algebra and Galois Theory

ORGANIZER: Zbigniew Hajto (*Jagiellonian University, Kraków, PL*)
Tuesday, July 3, 15:45–17:45, Seminar Hall

TALKS:

Askold Khovanskii (*University of Toronto, CA*), COAUTHORS: Yuri Burda, **Signatures of branched coverings**

Masa-Hiko Saito (*University of Kobe, JP*), **Geometry of moduli spaces of linear connections on curves and differential equations of Painlevé type**

Luis Narváez-Macarro (*University of Seville, ES*), **Symmetry properties of the roots of Bernstein-Sato polynomials and duality of D-modules**

Marius van der Put (*University of Groningen, NL*), **Stokes matrices for the quantum differential equations of some Fano varieties**

Signatures of branched coverings

Askold Khovanskii
University of Toronto, CA

A signature of a branched covering is the set of its branching points with the orders of local monodromy operators. The signatures can be classified as elliptic, parabolic and hyperbolic. In elliptic case the monodromy group of any covering with a given signature can be described explicitly. In the parabolic case the monodromy group can be described explicitly up to a commutative normal subgroup. In both cases there are many coverings with a given monodromy group. Finally in the hyperbolic case nothing can be said about the monodromy group: it can contain a subquotient isomorphic to an arbitrary finite group fixed in advance. As a corollary of this classification we get that all the algebraic equations and differential equations of Fuchs type with elliptic or parabolic signatures can be solved in radicals or quadratures respectively (except one case, when solution of an algebraic equation of degree 5 is required). My talk is based on a joint paper with Yuri Burda.

Coauthors: Yuri Burda

Geometry of moduli spaces of linear connections on curves and differential equations of Painlevé type

Masa-Hiko Saito
University of Kobe, JP

In this talk, I would like to give a short report on recent works on algebro-geometric approach of differential equations. The main topics are listed in the following.

1. Algebro-Geometric construction of moduli spaces of linear connection with fixed formal type of regular or irregular singularitieson a smooth projective curve by geometric invariant theory. Riemann-Hilbert correspondence and Painlevé property of isomonodromic differential equations. (Joint works of M. Inaba and K. Iwasaki and joint work of M. van der Put.)
2. Canonical coordinates on the moduli spaces of linear connections by apparent singularities (A joint work of S. Szabo)
3. Lagrangian fibrations on the moduli space of linear connections. (A joint work of F. Loray and C. Simpson)

Symmetry properties of the roots of Bernstein–Sato polynomials and duality of D-modules

Luis Narváez-Macarro
University of Seville, ES

In [G–S], Granger and Schulze have discovered and proved the symmetry about -1 of the roots of b-functions of reductive prehomogeneous determinants and of regular special linear free divisors. This symmetry property has been observed in many other examples and a general explanation is still missing. In this talk I will explain why the same symmetry is true for any free divisor of linear Jacobian type [NM]. The answer involves previous results on the duality of logarithmic D-modules [CM–NM]. On the other hand, the symmetry about $-\frac{n}{2}$ of the roots of reduced b-functions of isolated quasi-homogeneous singularities of hypersurfaces in dimension n is well known from the explicit knowledge of these roots. This motivates a conjecture, that I will state at the end, which tries to understand the above symmetries under the same scope.

References:
[CM–NM] F.J. Calderón Moreno and L. Narváez Macarro. "Dualité et comparaison sur les complexes de de Rham logarithmiques par rapport aux diviseurs libres". Ann. Inst. Fourier (Grenoble), 55(1) (2005), 47–75. "A mixed associativity formula for tensor products over two Lie–Rinehart algebras". Ann. Univ. Ferrara - Sez. VII - Sc. Mat., Vol. LI (2005), 105–118.
[G–S] M. Granger and M. Schulze. "On the Symmetry of b-Functions of Linear Free Divisors". Publ. RIMS Kyoto Univ., 46 (2010), 479–506.
[NM] L. Narváez Macarro. "A duality approach to the symmetry of Bernstein–Sato polynomials of free divisors". http://arxiv.org/abs/1201.3594

Stokes matrices for the quantum differential equations of some Fano varieties

Marius van der Put
University of Groningen, NL

This is joint work with John Alexander Cruz Morales. Associated to a Fano variety there is a Dubrovin–Givental connection which leads to a *quantum differential equation*. Dubrovin conjectured the values of the Stokes matrices of the quantum differential equation. In this lecture we will explain an explicit method for computing the Stokes matrices of some quantum differential equations. For the case of the projective spaces $\mathbb{P}^n$, Dubrovin's conjecture has been verified by Guzzetti. Our method provides an elementary proof of this and applies to more Fano varieties.