# Representation formulas and existence results in the theory of micropolar solid-fluid mixtures under steady state vibrations

Ionel-Dumitrel GHIBA dumitrel.ghiba@uaic.ro
*Department of Mathematics, "Al.I. Cuza" University of Iași,*
*Blvd. Carol I, no.11, 700506 Iași, Romania &*
*The Romanian Academy, "Octav Mayer" Mathematics Institute,*
*Blvd. Carol I, no.8, 700505 Iași, Romania*

## Abstract

A microcontinuum media, roughly speaking, is a continuum media whose properties and behaviour are affected by the local motions and deformations of the primitive elements. A special case of microcontinuum media is that of micropolar continua. In the micropolar continuum theory, the rotational degrees of freedom play a central role. Thus, we have six degrees of freedom, instead of three degrees of freedom considered in classical elasticity and fluid mechanics. A modern presentation, the state-of-art and the intended applications of these theories can be found in the books by Eringen (1999,2001). The higher-order or higher-grade continuum theories are necessary to capture size effects in small length scales, where the fundamental assumption of the classical continuum theory is that the (physical, chemical, mechanical, etc.) properties of a material are uniformly distributed throughout its volume fails. We remind that the theory of micropolar elastic materials has many applications concerning cellular solids.

On the other hand, many natural or synthetic materials are not pure materials. They are mixtures of two or more co-existence constituents. Sometimes the presence of a constituent can be ignored, if there is a preponderant constituent, but in many situations the local mechanical effects of each ingredient of the mixture cannot be ignored. In classical continuum theory, a mixture is idealized by assuming that every point in the mixture is occupied simultaneously by each constituent. A mixture is thereby envisioned as a superposition of several continuous media.

Taking into account the microstructural motions Twiss and Eringen introduce the mixture theory of materials with microstructure. In the last years many papers got back in discussion the study of mixtures with microstructure. Eringen (2003) has developed a continuum theory for a mixture of a micropolar elastic solid and a micropolar viscous fluid. This theory can be successfully applied to the study of engineering materials, as well as soils, rocks, granular materials, sand and underground water mixtures. Consolidation problems in the building industry, earthquake problems, oil exploration problems and cellular solids can be studied with the help of the mixture theories.

In various boundary-value problems from continuum mechanics it is important to give a representation of the general solution of the field equations in term of elementary (harmonic, biharmonic etc.) functions and to find the fundamental solution. In the present paper we consider the isothermal theory of a binary homogeneous mixture of an isotropic micropolar elastic solid with an incompressible micropolar viscous fluid. First, we establish a representation of Galerkin type for the dynamical problem. We use some Galerkin representations in order to determine the fundamental solutions for the three–dimensional problem governing the motion of a micropolar solid-fluid mixture in the case of steady state vibrations.

Then, we study the steady vibrations problem in the linear theory of micropolar solid-fluid mixture using the classical potential method. With the help of fundamental solutions of the steady vibration problem, we give representations of the Somigliana type. Inspired by these representations we define the potentials of single layer and double layer and then we use them to reduce the boundary value problems to singular integral equations. We prove that Fredholm's theorems are valid for these singular integral equations. Existence and uniqueness theorems are presented for interior and exterior problems. The method is a constructive one and can be used in order to obtain numerical solutions of the problems.

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