Liénard's polynomial system

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Abstract

We consider the general Liénard polynomial system with an arbitrary (but finite) number of singular points in the form

$$\dot{x} = y, \quad \dot{y} = -x\,(1+\beta_1\,x+\ldots+\beta_{2l}\,x^{2l})+y\,(\alpha_0+\alpha_1\,x+\ldots+\alpha_{2k}\,x^{2k}).$$

Applying a canonical system with field rotation parameters,

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x\,(1+\beta_1\,x \pm\, x^2+\ldots+\beta_{2l-1}\,x^{2l-1} \pm\, x^{2l}) \\ &\quad + y\,(\alpha_0+x+\alpha_2\,x^2+\ldots+x^{2k-1}+\alpha_{2k}\,x^{2k}), \end{aligned}$$

where $\beta_1,\, \beta_3, \ldots,\, \beta_{2l-1}$ are fixed and $\alpha_0,\, \alpha_2, \ldots,\, \alpha_{2k}$ are field rotation parameters, and using geometric properties of the spirals filling the interior and exterior domains of limit cycles, we prove the following theorem.

Theorem. *The general Liénard polynomial system can have at most $k+l$ limit cycles, k surrounding the origin and l surrounding one by one the other its singularities.*

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