Geometric and Quantitative Rigidity

Organizer: Marta Lewicka (*University of Pittsburgh, USA)*

Wednesday, July 4, 14:30–16:30, Conference Hall

Talks:

Daniel Faraco (*Madrid, ES*), **Families of quasiregular mappings and nonlinear elliptic systems**

Cristinel Mardare (*Paris 6, FR*), **Nonlinear Korn inequalities and existence of minimizers in nonlinear elasticity**

Caterina Ida Zeppieri (*Bonn University, DE*), **A rigidity estimate for fields with prescribed curl**

Patrizio Neff (*Universität Duisburg-Essen, DE*), Coauthors: Dirk Pauly, Karl-Josef Witsch, **A canonical extension of Korn's first inequality to H(Curl) motivated by gradient plasticity with plastic spin**

Families of quasiregular mappings and nonlinear elliptic systems

Daniel Faraco
Madrid, ES

Nonlinear Korn inequalities and existence of minimizers in nonlinear elasticity

Cristinel Mardare
Paris 6, FR

We show that the total energy of the pure displacement problem in nonlinear elasticity possesses a unique global minimizer for a large class of hyperelastic materials, including that of Saint Venant - Kirchhoff, provided the density of the applied forces are small in Lp-norm. The proof relies on a nonlinear Korn inequality with boundary showing that the H1-distance between two deformation fields is bounded, up to a multiplicative constant, by the L2-distance between their Cauchy-Green strain tensors.

A rigidity estimate for fields with prescribed curl

Caterina Ida Zeppieri
Bonn University, DE

Motivated by the study of nonlinear plane elasticity with dislocations, we prove that in dimension two the Friesecke, James and Müller rigidity estimate holds true also for fields β that are not curl-free, modulo an error depending on the mass of the $\operatorname{Curl}\beta$.

A canonical extension of Korn's first inequality to H(Curl) motivated by gradient plasticity with plastic spin

Patrizio Neff
Universität Duisburg-Essen, DE

We prove a Korn-type inequality in $\mathring{\mathsf{H}}(\mathrm{Curl},\Omega$ for tensor fields P mapping Ω to. More precisely, let $\Omega\subset\mathbb{R}^3$ be a bounded domain with connected Lipschitz boundary. Then, there exists a constant $c>0$ such that

$$C||P||_{\mathrm{L}^2(\Omega,)}\leq||\mathrm{sym}\ P||_{\mathrm{L}^2(\Omega,)}+||\mathrm{Curl}\ P||_{\mathrm{L}^2(\Omega,)} \qquad (0.1)$$

holds for all tensor fields $P\in\mathring{\mathsf{H}}(\mathrm{Curl},\Omega$, i.e., all $P\in\mathsf{H}(\mathrm{Curl},\Omega,)$ with vanishing tangential trace on . Here, rotation and tangential trace are defined row-wise. For compatible P, i.e., $P=\nabla v$ and thus $\mathrm{Curl}P=0$, where $v\in\mathsf{H}^1(\Omega,\mathbb{R}^3)$ are vector fields having components v_n, for which ∇v_n are normal at , the presented estimate (0.1) reduces to a non-standard variant of Korn's first inequality, i.e.,

$$c||\nabla v||_{\mathrm{L}^2(\Omega,)}\leq||\mathrm{sym}\nabla v||_{\mathrm{L}^2(\Omega,)}.$$

On the other hand, for skew-symmetric P, i.e., $\mathrm{sym}P=0$, (0.1) reduces to a non-standard version of Poincaré's estimate. Therefore, since (0.1) admits the classical boundary conditions our result is a common generalization of the two classical estimates, namely Poincaré's resp. Korn's first inequality.
Key Words Korn's inequality, gradient plasticity, theory of Maxwell's equations, Helmholtz decomposition, Poincaré/Friedrichs type estimate

Coauthors: Dirk Pauly, Karl-Josef Witsch