Neugebauer lecture: Mathematicians and Decorative Geometric Tilings in the Medieval Islamic World

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Otto Neugebauer (1899-1990): personal memories (1983-1984)

Wilbour Hall, Brown University

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Neugebauer's History of Mathematics Department at Brown

Abe Sachs (1915-1983), assyriologist

David Pingree (1933-2005), specialist in Sanskrit

Gerald Toomer (born 1934), classicist

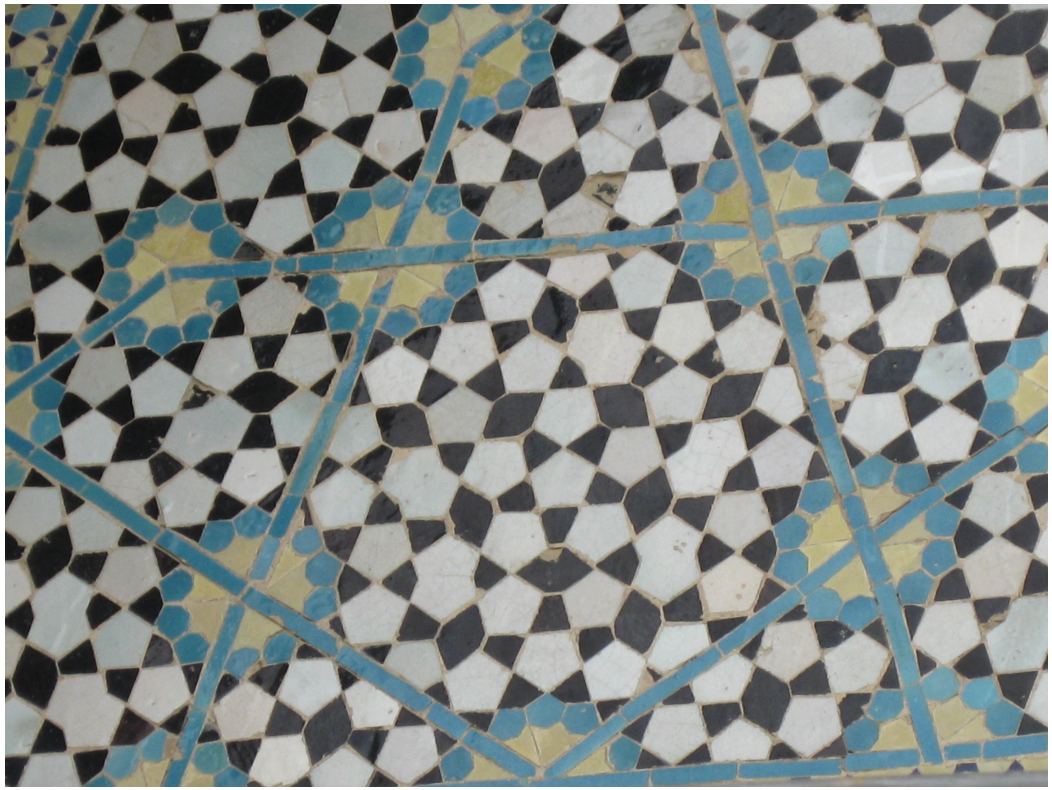
The Neugebauer School in History of Mathematics until ca. 1600

1. Use manuscript sources in many languages; Akkadian (cuneiform), Arabic, Persian, Hebrew, Sanskrit, Ethiopean, etc. (Greek and Latin by default)

2. Study mathematics in contexts: astronomy, astrology, etc.

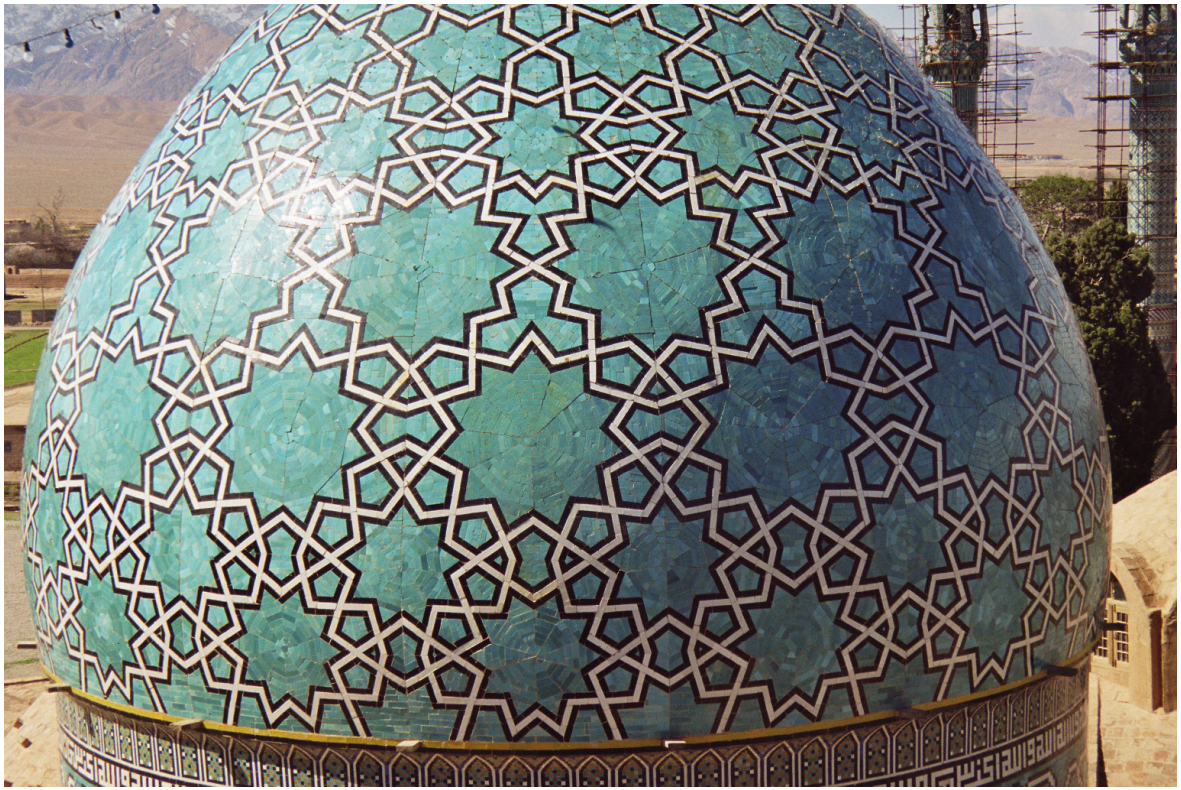
3. Technical analysis of sources leads to historical conclusions on transmission of mathematical knowledge between cultures.

‘Mathematical art’ in medieval Islamic civilization (ca. 1050-1750): 1. Plane tilings



Darb-e Imam, Isfahan, Iran (15th c.)

2. Tiled cupolas



Shrine of Shah Nematollah Vali, Mahan, Iran (15th c.)

3: Three-dimensional art: Muqarnas

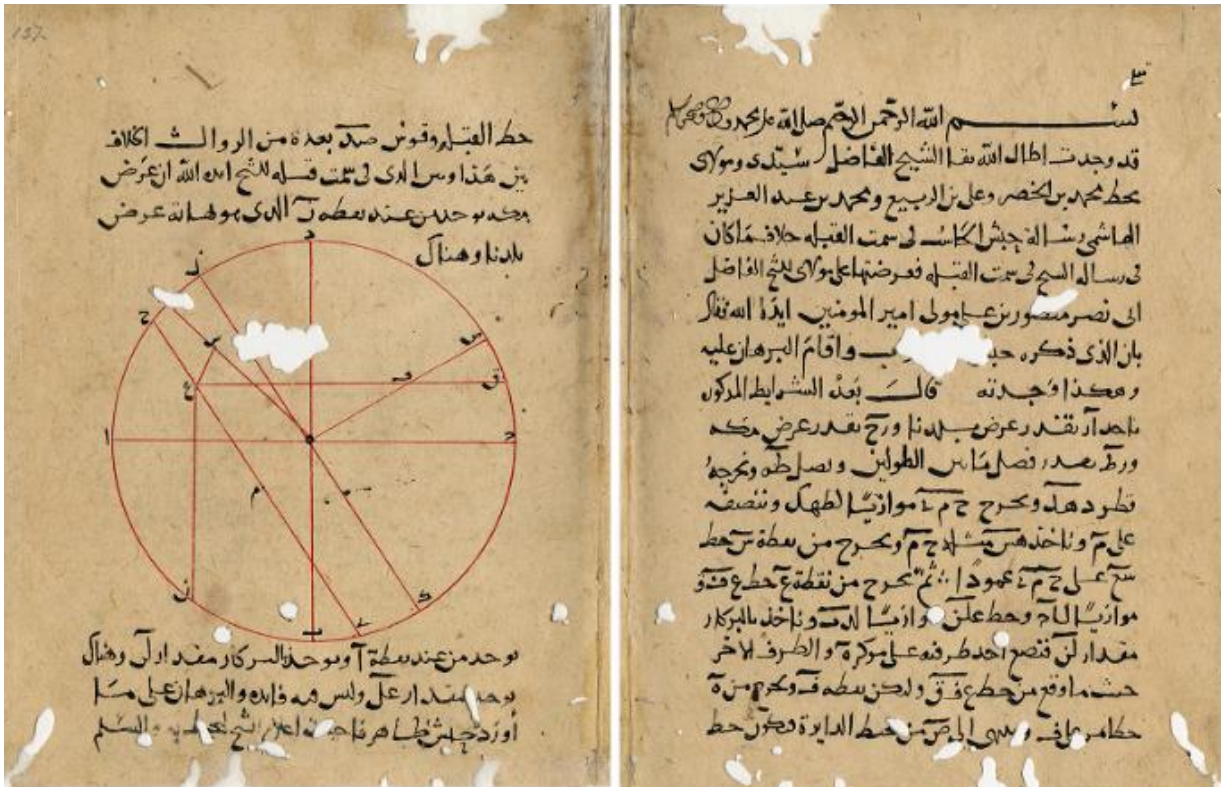
Shiraz, Iran

Questions

What mathematical methods, if any, were used in the design and construction of the plane tilings?

What was the interaction, if any, between the designers and the Islamic mathematician-astronomers?

Hundreds of Arabic and Persian manuscripts on mathematics have been preserved



Most of the texts were written by mathematicians-astronomers, trained in Greek (deductive) mathematics. They contain almost no information on tilings.

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Who designed and constructed tilings?

Not mathematicians-astronomers but craftsmen (Arabic: *ṣunnāᶜ*).

Sources are extremely scarce and fragmentary. We will discuss:

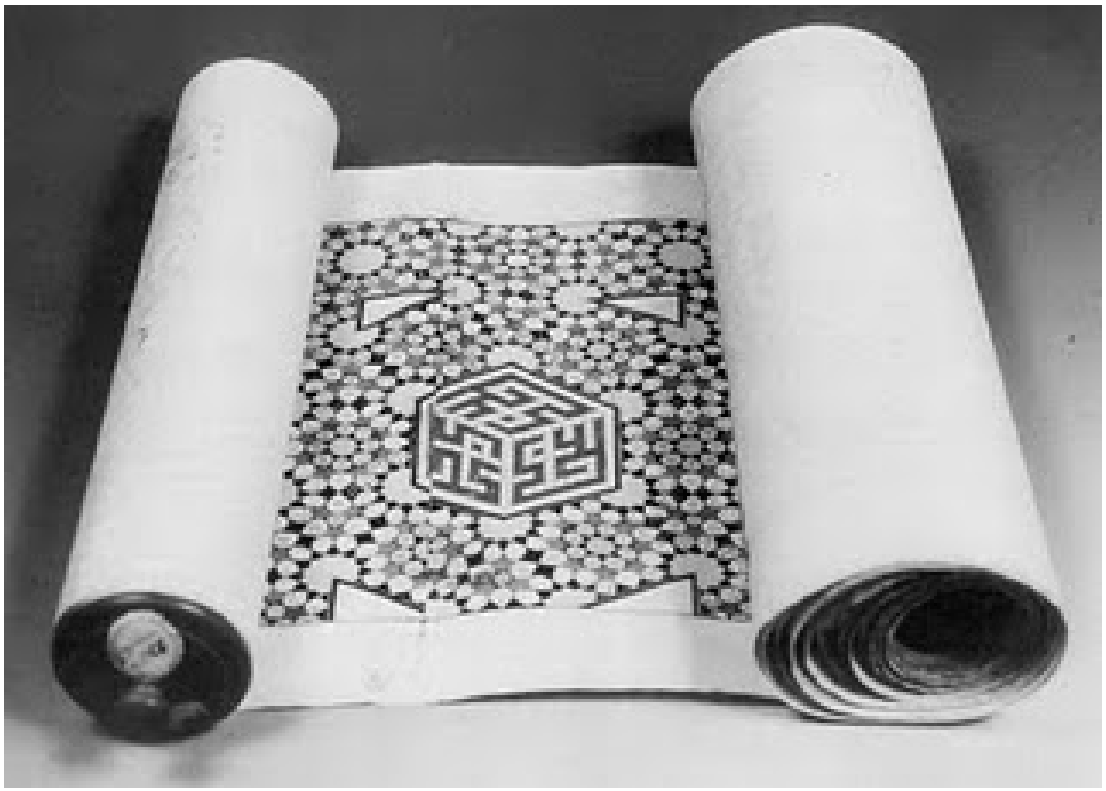
1. Abu'l-Wafā's booklet (ca. 990) on the geometry necessary for craftsmen (he was a mathematician-astronomer)
2. The Topkapi Scroll, a document by or for craftsmen
3. Anonymous medieval Persian manuscript, by or for craftsmen
4. A work by a mathematician (Omar Khayyām, 1048-1131) on a problem inspired by craftsmen.

1. Abu'l-Wafā' (10th-c.): booklet on the geometry necessary for craftsmen

He says: Craftsmen (*ṣunnāᶜ*) use geometrical constructions but no proofs, and do not distinguish between exact and approximate constructions.

They used ruler, compass (sometimes with a fixed opening), and a set-square ("gonia").

2. The Topkapi Scroll: a document by/for craftsmen



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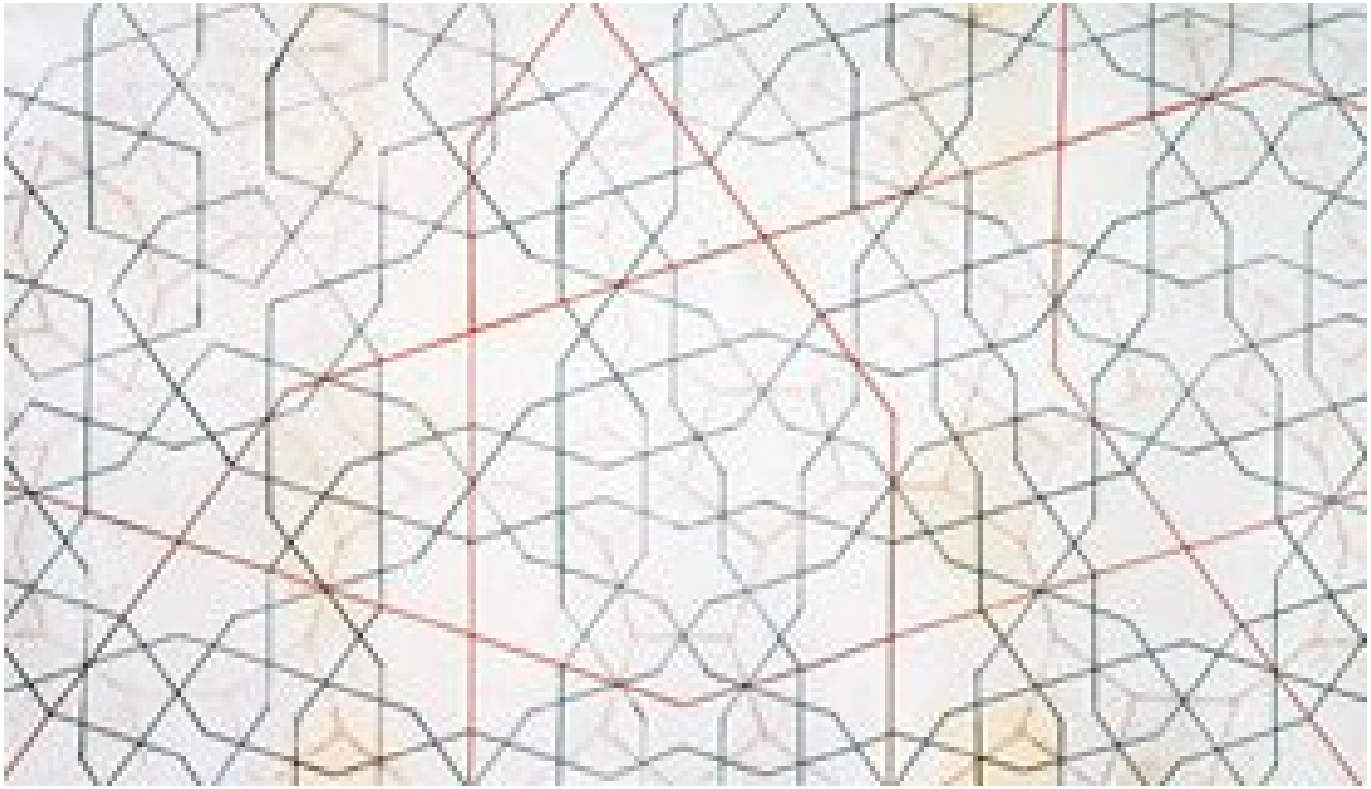
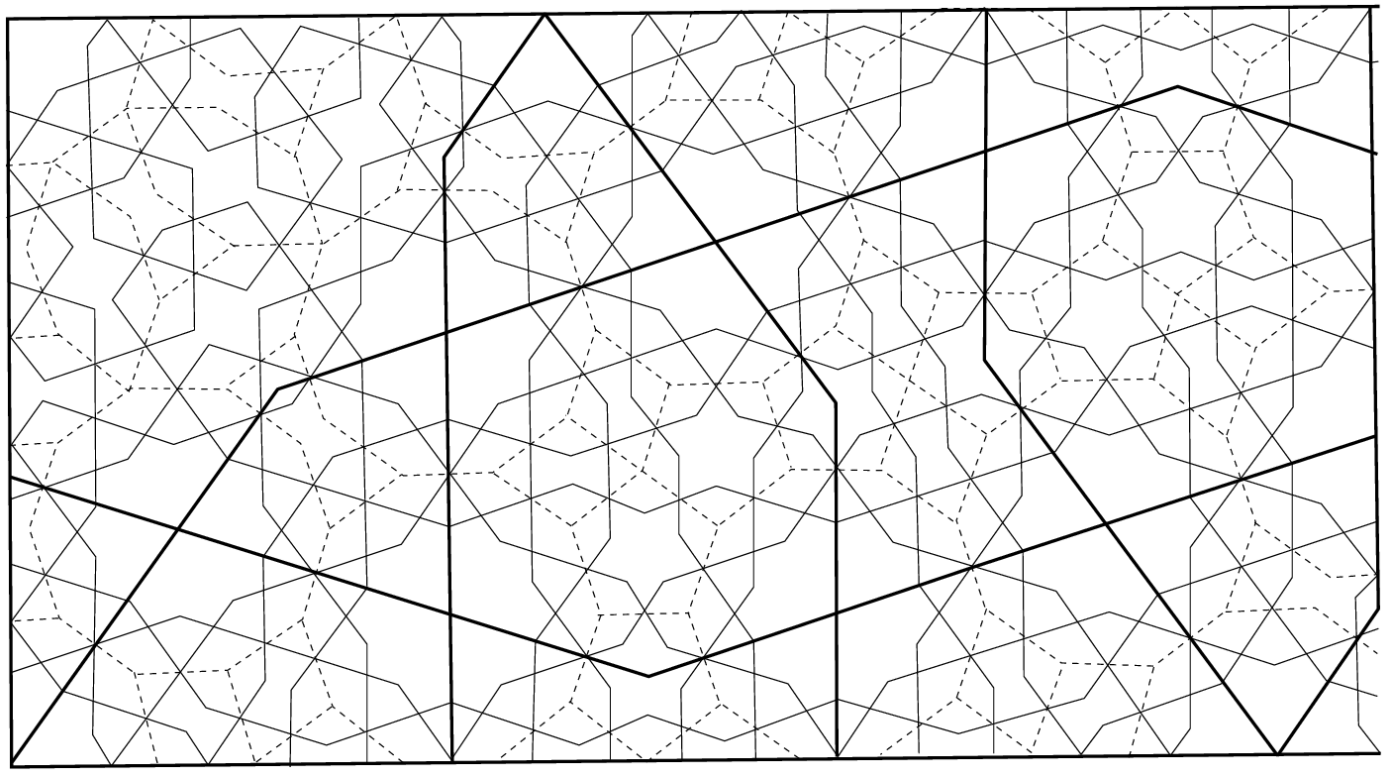
Topkapi Scroll: a diagram



The dotted figures ("gireh tiles") were probably used in the design process



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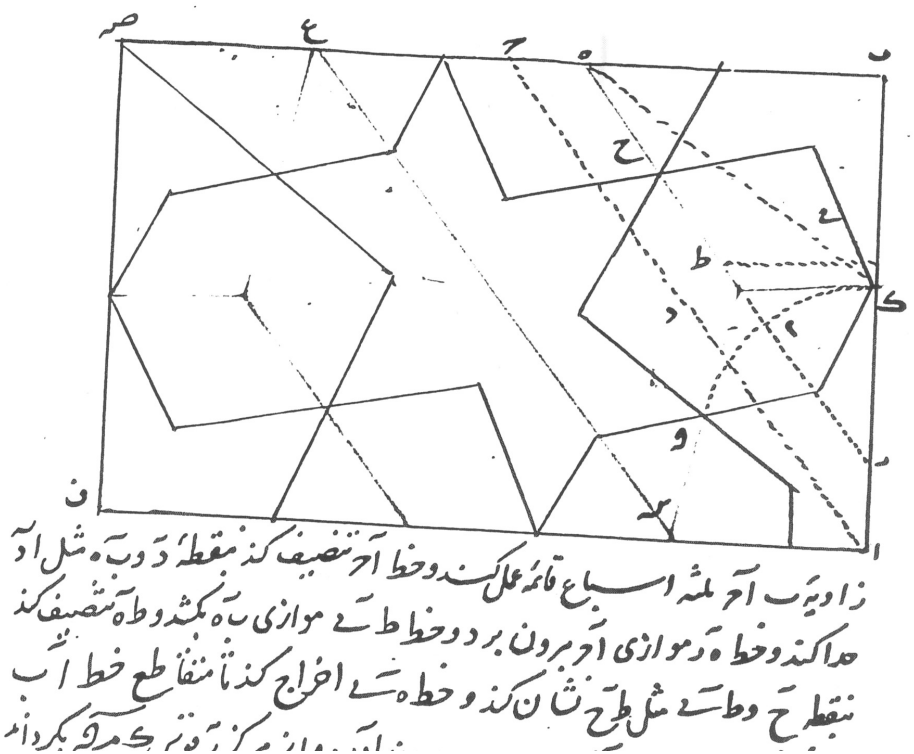
3. Anonymous 16th c (?) Persian manuscript, Paris, Bibliothèque Nationale, Or. 169



40 pages, to be published in 2013.

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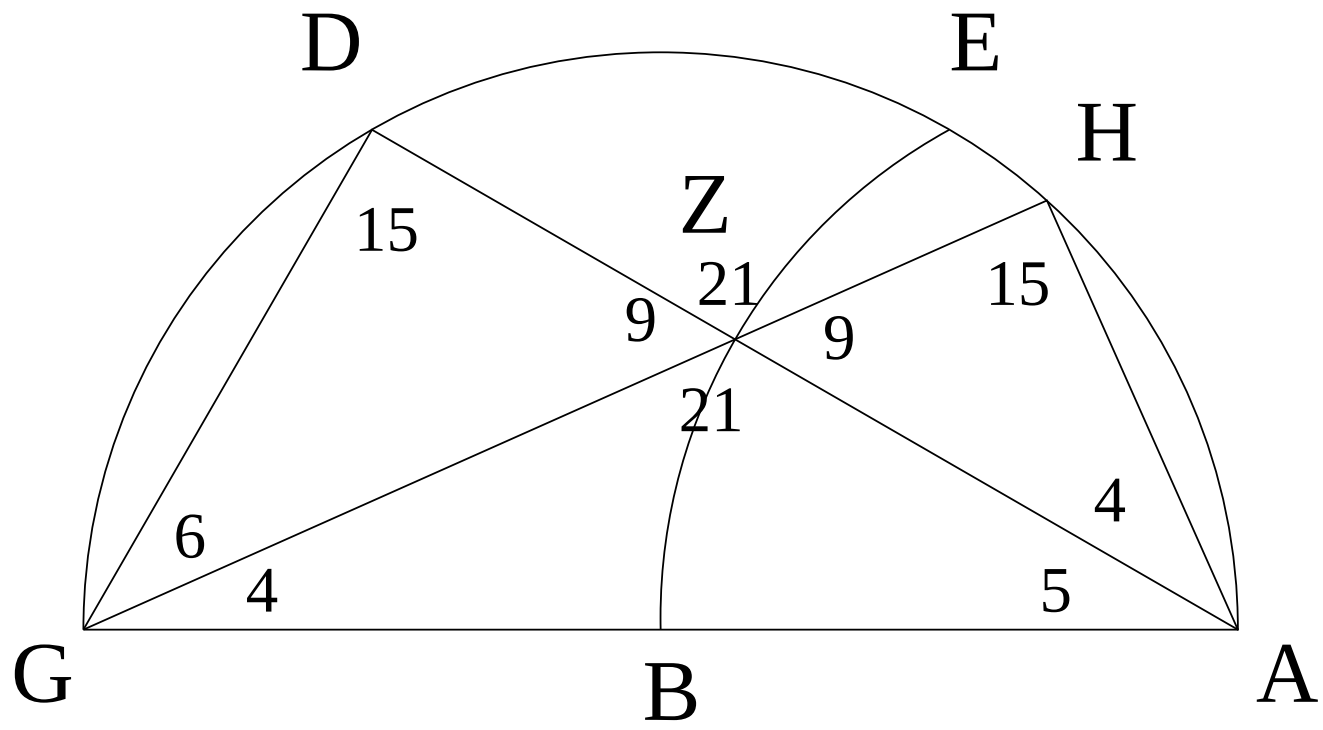
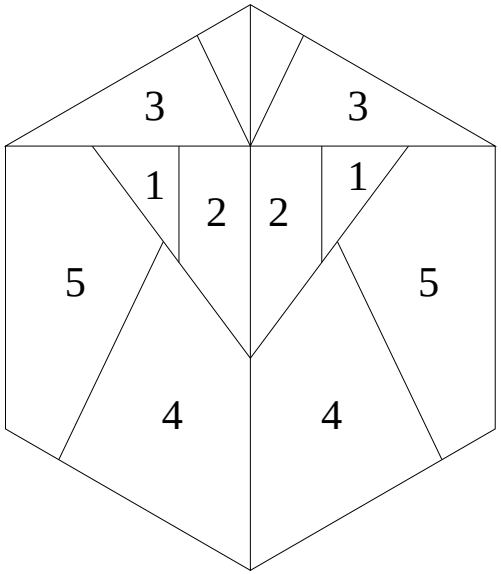
Persian manuscript: 3.1. example of approximate construction with fixed compass-opening

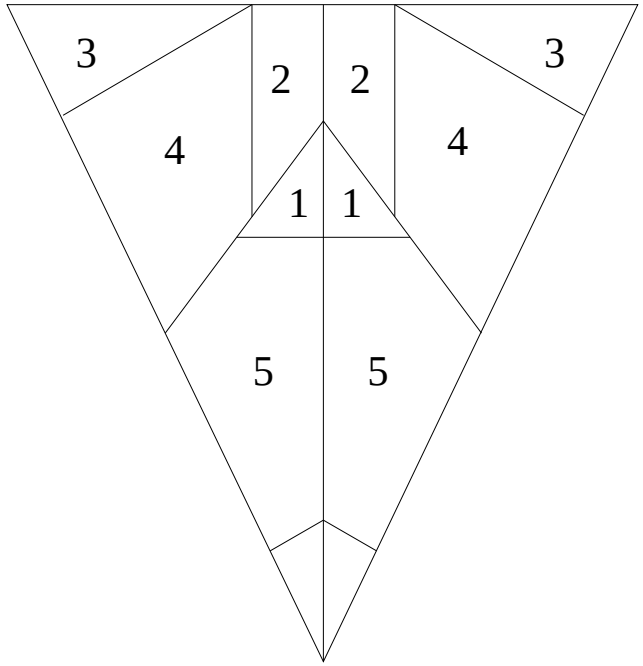
Construction of side of pentagon (DH) inscribed in the circle with center B and circumscribing set-square ADG with angles $30^o, 60^o, 90^o$ (in modern terms)



manuscript has this figure with brief explanation

Persian manuscript: 3.2. example of cut-and-paste construction





Only figures, no text

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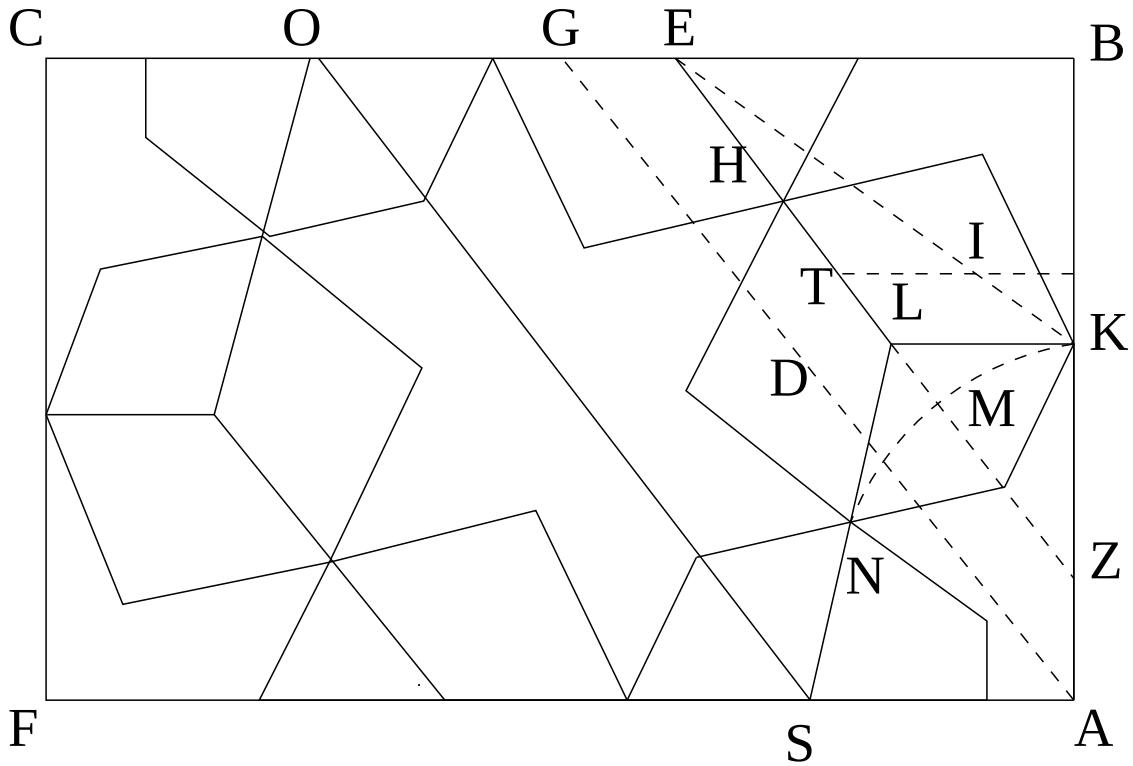
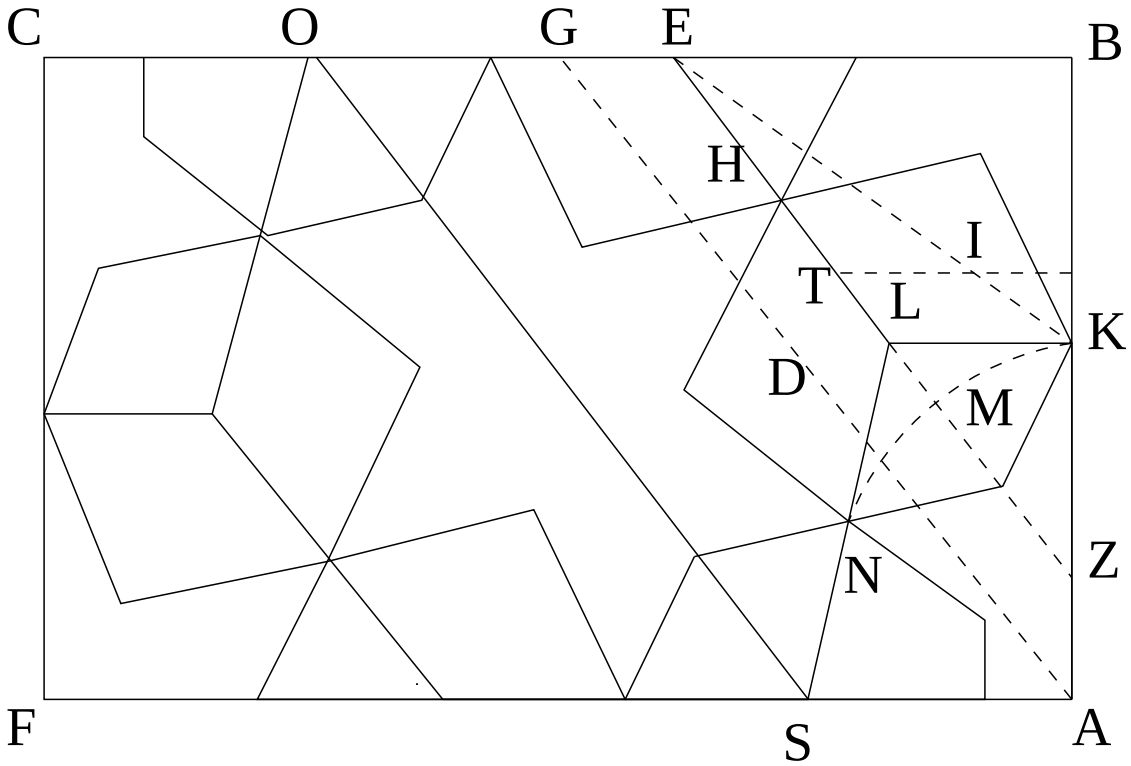
Persian manuscript: 3.3. example of tiling with explanation



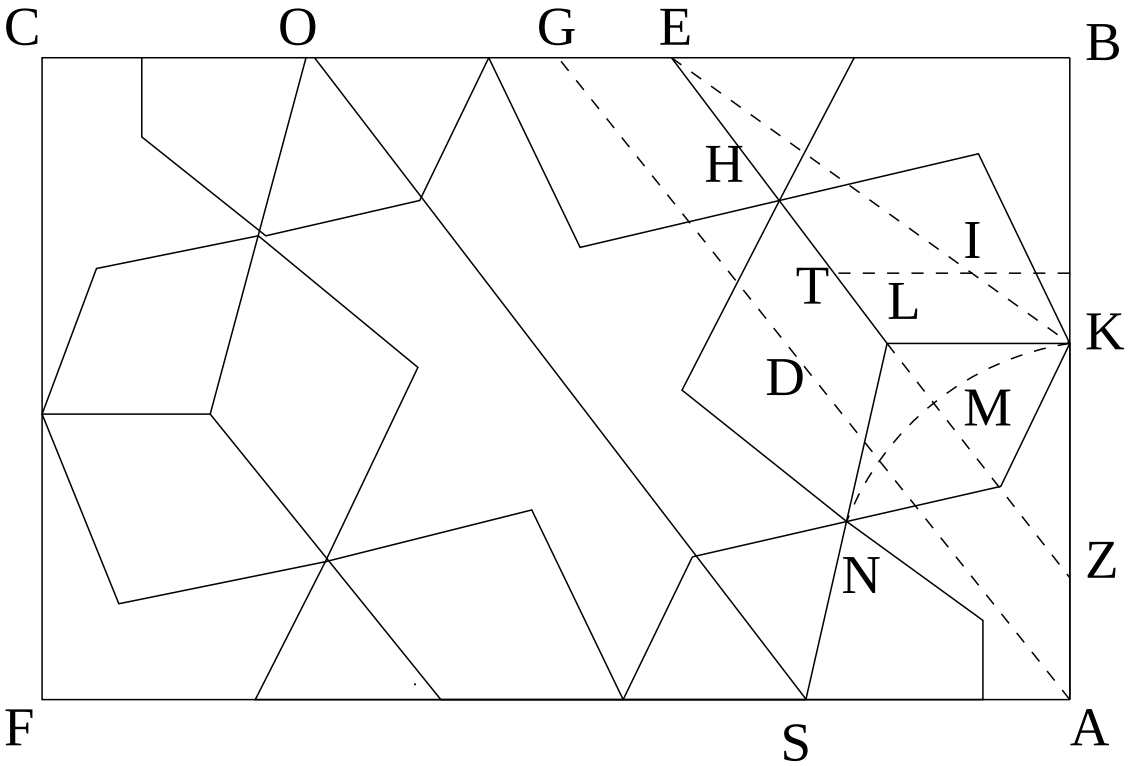
"Make angle *BAG* three sevenths of a right angle. Bisect *AG* at point *D*. Cut off *BE* equal to *AD*. Produce line *EZ* parallel to *AG*."

Explanation (continued)



"Draw line *TI* parallel to *BE*, bisect *TE* at point *H*, and make *TI* equal to *TH*. Extend *EI* until it intersects *AB* at point *K*."

Explanation (continued)



"Produce *KL* parallel to *BE*. With center *Z* draw circular arc *KMN* in such a way that its part *KM* is equal to *MN*. On line *AF* take point *S* and that is the center of a heptagon. Complete the construction, if God Most High wants."

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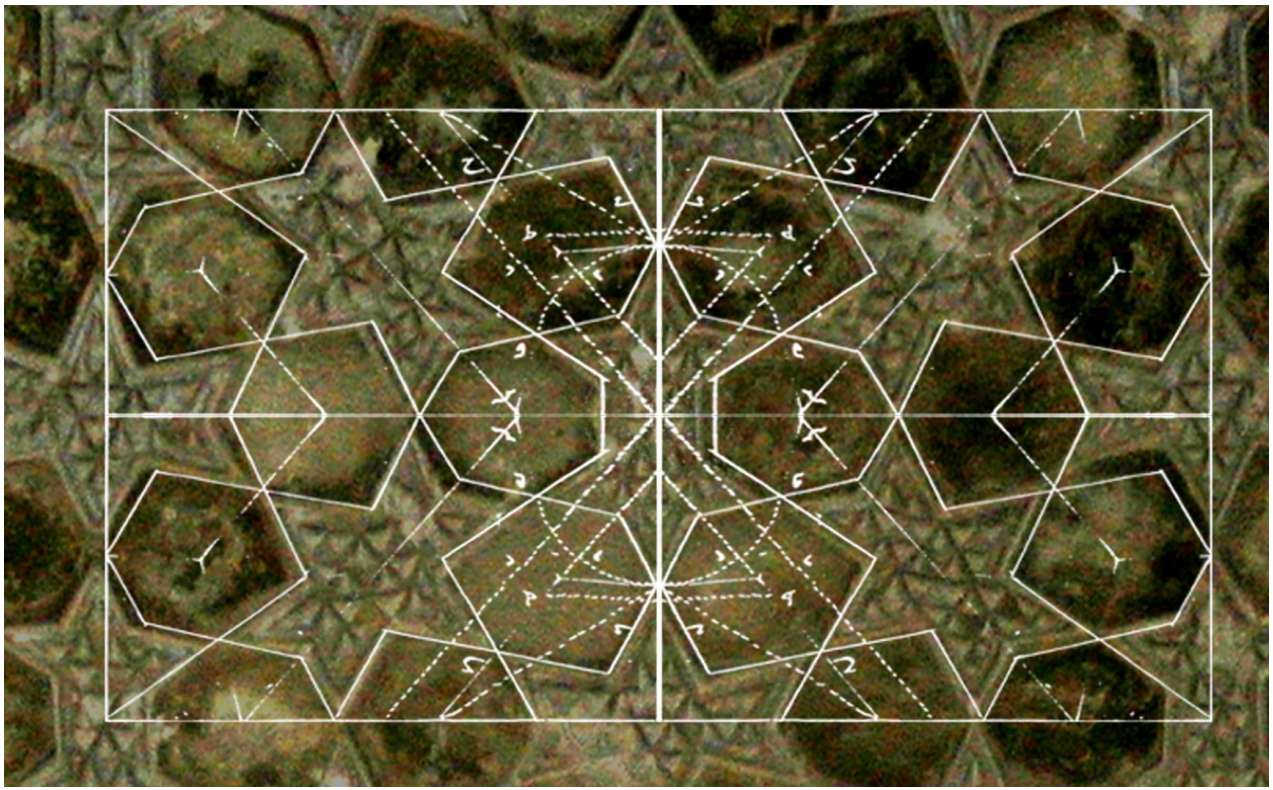
Tiling realized in Isfahan, North Cupola of the Friday Mosque (ca. 1080)



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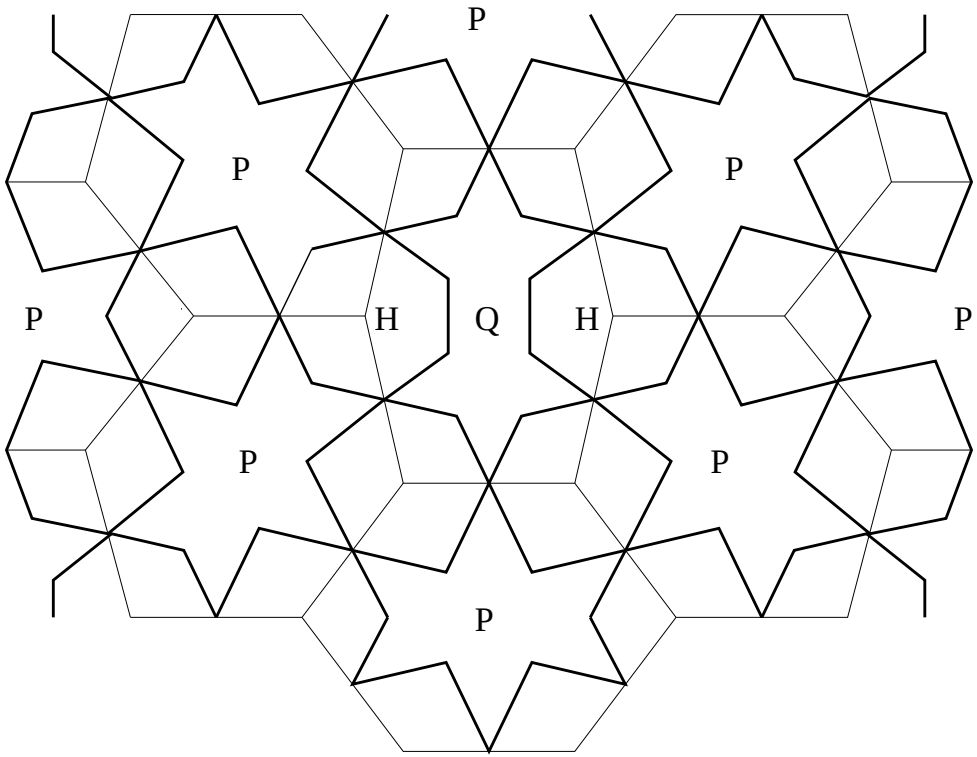
The putting together of the tiling is not explained in the Persian manuscript



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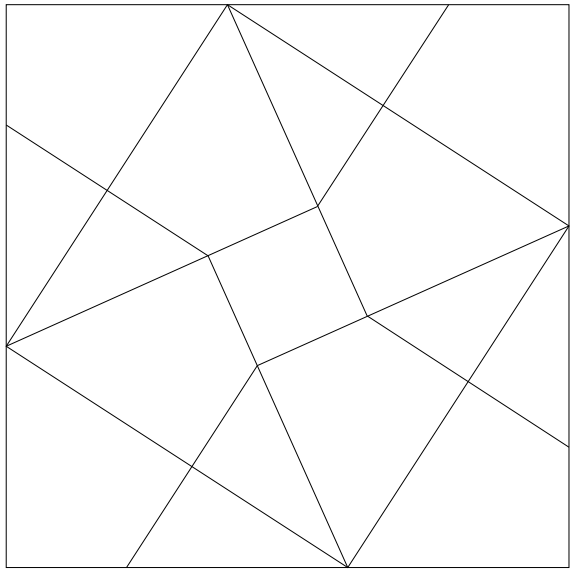
Persian manuscript 3.4: Twelve Kite Pattern

Four kite pattern
Isfahan, Hakim mosque



Twelve kite pattern in Persian manuscript:
4 big kites, each subdivided into 2 smaller kites and 2 triangles.

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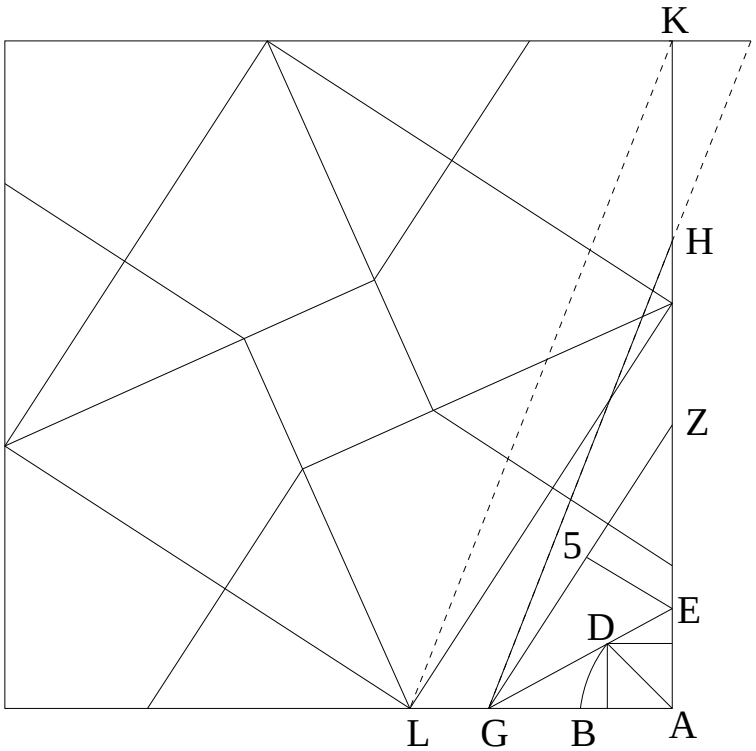
Approximative construction of Twelve Kite Pattern in the Persian manuscript



Begin with little square,
AD diagonal,
$GB = BA = AD$,
$EZ = ZH = AG$,
$KL//HG$.

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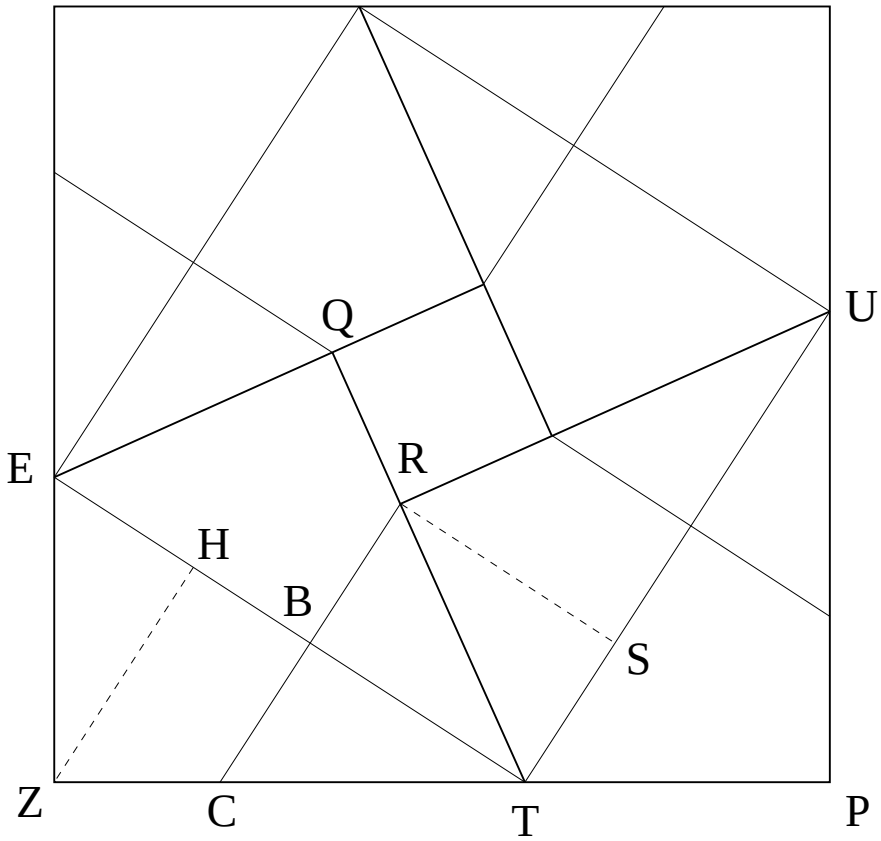
The Twelve Kite Pattern and a special right angled triangle (EZT), also in the Persian manuscript.



altitude plus shortest side is hypotenuse ($EZ + ZH = ET$)

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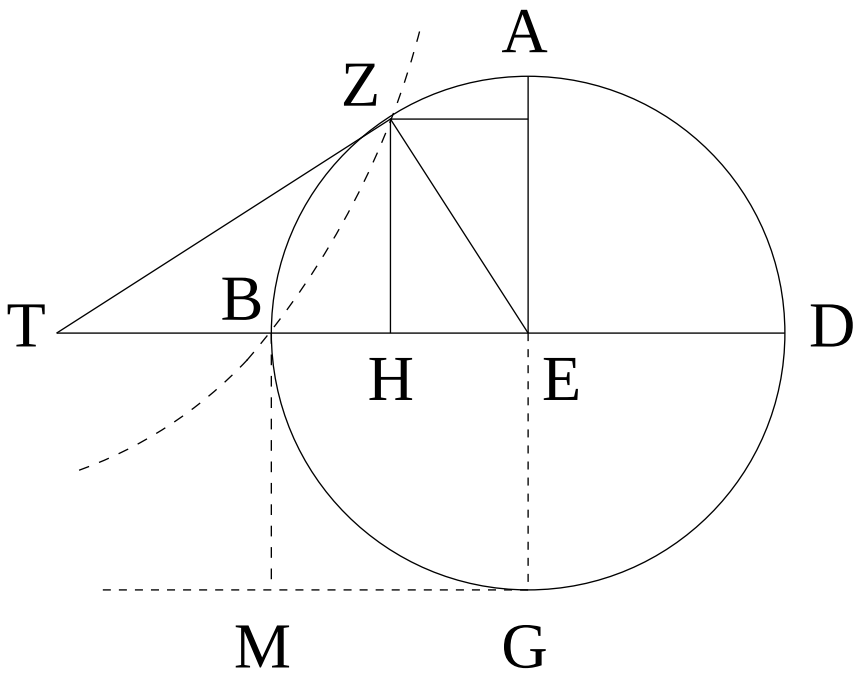
4. Mathematicians and the Twelve Kite Pattern. 4.1 Omar Khayyam in a letter on algebra (ca. 1100)

To Construct a right-angled triangle such that: altitude + shortest side = hypotenuse
(EZT such that $EZ + ZH = ET$, notation mine.)

If $EH = 10$ and ZH="thing" (modern: x) then
"a cube and two hundred things are equal to twenty squares plus two thousand in number"
modern $x^3 + 200x = 20x^2 + 2000$.
segment with length x can be constructed by means of conic sections.

4.2 Anonymous construction of the triangle (EZT) by means of conic sections, in an appendix to Omar Khayyam's text



$EZ + ZH = ET$.

Circle, center E, two perpendicular diameters AG, BD.
Hyperbola through B, asymptotes AG and line through G parallel to BD.
Z point of intersection

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Conclusions 1: the mathematics of craftsmen

This type of mathematics rarely appears in the works of the mathematician-astronomers.

The craftsmen used geometric constructions without proofs, made no distinction between approximate and exact; used ruler, set-square, compass (with fixed opening), other instruments as well.

Their approximate constructions were often very accurate

Oral instruction must have been important

Conclusions 2: Interaction between mathematicians and craftsmen - as far as we can judge

a. Abu'l-Wafa criticised the craftsmen

b. Omar Khayyam and other mathematicians were inspired by the Twelve Kite Pattern

c. The craftsmen respected the work of the mathematicians but did not understand the details.

Appendix: Destination of the Neugebauer Prize

Continued collaboration, including student exchange, with the House of Mathematics, Isfahan (www.mathhouse.org)

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Mathematics and Art Conference, Isfahan, 2006

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Students and teachers from Utrecht, Leiden and Isfahan, 2006

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Workshop on Mathematics and Dutch and Islamic art, Isfahan, 2006

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Reviving the old crafts in a new building, Isfahan, House of Mathematics

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Thank you!

This presentation can be downloaded at

http://www.jphogendijk.nl/talks/neugebauer.pdf

A written version of this talk can be found on

http://www.jphogendijk.nl/talks/neugebauer-written.pdf

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