Hyperbolic Conservation Laws

ORGANIZERS: Piotr Gwiazda (*University of Warsaw, PL)*, Agnieszka Świerczewska-Gwiazda (*University of Warsaw, PL)*

Wednesday, July 4, 14:30–16:30, Medium Hall A

TALKS:
Boris Andreianov (*Université de Franche-Comté, Besançon, FR*), **Conservation laws with discontinuous flux**

Miroslav Bulíček (*Charles University, Prague, CZ*), **On scalar hyperbolic laws with discontinuous fluxes**

Philippe G. LeFloch (*Université Pierre et Marie Curie (Paris 6), FR*), **Compressible fluid flows with finite energy**

Athanasios E. Tzavaras (*University of Crete, GR*), **The equations of polyconvex elasticity and the problem of dynamic cavitation**

Conservation laws with discontinuous flux

Boris Andreianov boris.andreianov@univ-fcomte.fr
Université de Franche-Comté, Besançon, FR

The mainstream mathematical theory for scalar hyperbolic conservation laws, such as the inviscid Burgers equation, is more than 40 years old. The case of conservation laws with discontinuous in space

$$u_t + (\mathfrak{f}(x,u))_x = 0, \quad \mathfrak{f}\colon (x,z) \in \mathbb{R}\times\mathbb{R} \mapsto \begin{cases} f^l(z) & x<0, \\ f^r(z) & x>0. \end{cases} \tag{1}$$

was a subject of intense research since almost twenty years. It is known, for instance, that infinitely many consistent (and physically motivated) solution semigroups may co-exist; that a straightforward adaptation of the classical approach may fail ; that the numerical schemes such as Godunov or wave-front tracking converge, though it is not always easy to construct the Riemann solver compatible with a given physical approximation procedure (such as a vanishing viscosity or a vanishing capillarity).

In this talk, we present a theory for problem (1) that brings a unified approach to well-posedness and numerical approximation of discontinuous-flux problems. Roughly speaking, we introduce the concepts that describe the interface coupling and put forward the techniques to deal with the interface within the appropriately re-interpreted approach of Kruzhkov. To motivate the advantage of our approach, we will briefly describe the applications of the theory to the case of flow in layered porous medium.

The talk is based on joint works with K.H. Karlsen and N.H. Risebro (the general theory) and with N. Seguin, C. Cancés, and P. Goatin (applications).

On scalar hyperbolic laws with discontinuous fluxes

Miroslav Bulíček
Charles University, Prague, CZ

We consider conservation laws with a flux that can have jump discontinuities in an unknown function. We introduce new concepts of entropy weak and measure-valued solution that are consistent with the standard ones if the flux is continuous. Having various definitions of solutions to the problem, we then answer the question what kind of properties the flux should possess in order to establish the existence and/or uniqueness of solution of a particular type. In any space dimension we establish the existence of measure-valued entropy solution for a flux having countable jump discontinuities. Under the additional assumption on the Hlder continuity of the flux at zero, we prove the uniqueness of entropy measure-valued solution, and as a consequence, we establish the existence and uniqueness of weak entropy solution. Finally, we extend the theory also on a class of uxes that are also x-dependent.

Compressible fluid flows with finite energy

Philippe G. LeFloch http://philippelefloch.org
Université Pierre et Marie Curie (Paris 6), FR

We consider the Euler equations describing the dynamics of compressible (real) fluid flows in one space dimension governed by a general equation of state, and we establish existence and compactness results for weak solutions satisfying entropy inequalities. We encompass solutions with finite energy that possibly have large amplitude and contain vacuum states, and our framework for instance yields the existence of radially symmetric solutions defined in the physical space including the coordinate singularity at the center. Higherintegrability properties of entropy solutions are established and TartarŠs commutation relation for Young measures with unbounded support is analyzed here. The mathematical entropy pairs are described by a kernel which has limited regularity on the boundary of its support and this limited regularity leads one to analyze certain singular products.

The equations of polyconvex elasticity and the problem of dynamic cavitation

Athanasios E. Tzavaras tzavaras@tem.uoc.gr
University of Crete, GR

The equations of elastodynamics are a paradigm of a system of conservation laws where the lack of uniform convexity of the stored energy function poses challenges in the mathematical theory.

First, we show how the existence of certain nonlinear transport identities reinforces the efficacy of the entropy as a stabilizing factor and recovers the theory associated wth uniformly convex entropies in hyperbolic systems. The equations of elastodynamics with polyconvex stored energy can be embedded into a larger symmetric hyperbolic system and visualized as constrained evolution. This leads to a variational approximation scheme and an existence theory for dissipative measure valued solutions. Moreover, when a smooth solution is present, it is unique within the class of dissipative measure valued solutions.

Second, we focus on the system of radial elastodynamics for isotropic elastic materials. We will recall a non-uniqueness result due to S. Spector and K. Pericak-Spector and associated with the onset of cavitation. We analyze this counterexample from the perspective of a theory that accounts for the singular layers of the cavitating solution.