Implicitly Constituted Material Models: Modeling and Analysis

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Organizers: Josef Málek (*Charles University, CZ)*, Endre Süli (*University of Oxford, UK)*

Tuesday, July 3, 15:45–17:45, Conference Hall

Talks:
Vít Průša (*Charles University, Prague, CZ*), **An introduction to implicit theories to describe the response of bodies**

Jens Frehse (*University of Bonn, DE*), **On Prandtl-Reuss mixtures**

Miroslav Bulíček (*Charles University, Prague, CZ*), **On the analysis of unsteady flows of implicitly constituted incompressible fluids**

Endre Süli (*University of Oxford, UK*), **Existence of global weak solutions to implicitly constituted kinetic models of incompressible homogeneous dilute polymers**

An introduction to implicit theories to describe the response of bodies

Vít Průša prusv@karlin.mff.cuni.cz
Charles University, Prague, CZ

Most of the popular mathematical models to describe the response of bodies are ex- plicit models in the sense that the stress is expressed explicitly in terms of an appropriate kinematical variable or vice-versa; the classical linearized elastic model, the NavierŰStokes fluid model, the classical linear(ized) viscoelasic fluid model, the generalized Stokesian fluid model (including power-law fluids), and RivlinŰEricken fluids of the differential type are some examples. The class of simple materials is thought of as being a very general mate- rial model. However, many of the phenomena exhibited by bodies cannot be adequately described by such explicit models. Recently developed implicit constitutive theory is, in a sense, based on the simple idea of expressing the response of bodies by an implicit relation between the stress and appropriate kinematical variables. Such approach appears to be capable of describing some of the material properties that explicit models seem unable to describe. As in the classical case the arising governing equations are systems of nonlinear partial differential equations, but the implicit nature of the models leads to a non-standard structure of these equations. Naturally, the equations deserve mathematical investigation, and provide a rich source of interesting mathematical problems. In the talk we will, following the work of Rajagopal and coauthors, discuss the develop- ment of the implicit constitutive theory and its thermodynamical underpinnings. We will also introduce some simple models that fit into the new framework and that do not have classical counterparts.

On Prandtl-Reuss mixtures

Jens Frehse mathfrehse@googlemail.com
University of Bonn, DE

We consider the analogue of the Prandtl–Reuss law for a mixture of a hard and soft material. This model was developed by Rajagopal and his co-authors. The purpose of this model is to try to provide physical explanation of elastic plastic deformation with hardening. The role of the internal parameters is replaced by the partial stress given by the hard material.

We prove that the partial strains and stresses have the same degree of regularity in Lebesgue-, Sobolev-, and Besov-spaces that are known in the classical case.

On the analysis of unsteady flows of implicitly constituted incompressible fluids

Miroslav Bulíček mbul8060@karlin.mff.cuni.cz
Charles University, Prague, CZ

The talk will survey recent joint work with Piotr Gwiazda, Josef Málek and Agnieszka Świerczewska-Gwiazda. We consider incompressible fluids with a general algebraic implicit constitutive equation relating the deviatoric part of the Cauchy stress $\mathbf{S}$ and the symmetric part of the velocity gradient $\mathbf{D}$ in such a way that it leads to a maximal monotone (possibly multivalued) graph and the rate of dissipation is characterized by the sum of a Young function depending on $\mathbf{D}$ and its conjugate being a function of $\mathbf{S}$. Such a framework is very robust and includes, among others, classical power-law fluids, stress power-law fluids, fluids with activation criteria of Bingham or Herschel–Bulkley type, and shear-rate-dependent fluids with discontinuous viscosities as special cases. The appearance of $\mathbf{S}$ and $\mathbf{D}$ in all the assumptions characterizing the implicit relationship $\mathbf{G}(\mathbf{D},\mathbf{S}) = \underline{0}$ is fully symmetric. We present new long-time and large-data existence results for a very general class of parameters.

Bibliography

M. Bulíček, P. Gwiazda, J. Málek, and A. Świerczewska-Gwiazda. On Unsteady Flows of Implicitly Constituted Incompressible Fluids. *SIAM J. Math. Anal.*, 2012. (Submitted for publication).

Existence of global weak solutions to implicitly constituted kinetic models of incompressible homogeneous dilute polymers

Endre Süli endre.suli@maths.ox.ac.uk
University of Oxford, UK

The talk will survey recent joint work with Miroslav Bulíček and Josef Málek at the Mathematical Institute, Faculty of Mathematics and Physics, Charles University, Prague.

We show the existence of global weak solutions to a general class of kinetic models of homogeneous incompressible dilute polymers. The main new feature of the model is the presence of a general implicit constitutive equation relating the viscous part $\mathbf{S}_v$ of the Cauchy stress and the symmetric part $\mathbf{D}$ of the velocity gradient. We consider a general class of implicit relations that generate maximal monotone (possibly multivalued) graphs, and the corresponding rate of dissipation is characterized by the sum of a Young function and its conjugate depending on $\mathbf{D}$ and $\mathbf{S}_v$, respectively. The appearance of $\mathbf{S}_v$ and $\mathbf{D}$ in all the assumptions characterizing the implicit relationship $\mathbf{G}(\mathbf{S}_v, \mathbf{D}) = \underline{0}$ is fully symmetric. The elastic properties of the flow, characterizing the response of polymer macromolecules in the viscous solvent, are modelled by the elastic part $\mathbf{S}_e$ of the Cauchy stress tensor, whose divergence appears on the right-hand side of the momentum equation, and which is defined by the Kramers expression involving the probability density function, associated with the random motion of the polymer molecules in the solvent. The probability density function satisfies a Fokker–Planck equation, which is nonlinearly coupled to the momentum equation.

We establish long-time and large-data existence of weak solutions to such a system, completed by an initial condition and either a no-slip or Navier's slip boundary condition, by using properties of maximal monotone operators and Lipschitz approximations of Sobolev-space-valued Bochner functions via a weak compactness argument based on the Div-Curl Lemma and Chacon's Biting Lemma. A key ingredient in the proof is the strong compactness in L^1 of the sequence of Galerkin approximations to the probability density function and of the associated sequence of approximations to the elastic part $\mathbf{S}_e$ of the Cauchy stress tensor.

Bibliography

M. Bulíček, J. Málek, and E. Süli. Existence of global weak solutions to implicitly constituted kinetic models of incompressible homogeneous

dilute polymers. *Comm. Partial Differential Equations*, 2012. (Submitted for publication).