Infinite-dimensional Dynamical Systems with Time Delays

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Thursday, July 5, 16:15–18:15, Seminar Hall

TALKS:

Hans-Otto Walther (*Universitat Giessen, DE*), **Evolution systems for diőerential equations with variable time lags**

Bernhard Lani-Wayda (*University of Giessen, DE*), **Hopf bifurcation for retarded FDE and for semiflows**

Tibor Krisztin (*University of Szeged, HU*), **The attractor of slow oscillation for delayed negative feedback**

Ferenc Hartung (*University of Pannonia, Veszprém, HU*), **Smooth dependence on parameters of solutions in functional differential equations with state-dependent delays**

Evolution systems for diőerential equations with variable time lags

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We begin with the construction of semiflows for autonomous delay differential equations of the general form $x'(t) = g(x_t)$, which include equations with unbounded state-dependent delays. These semiflows consist of continuously differentiable solution operators which are defined on suitable Banach manifolds of differentiable functions. At equilibria we obtain local stable and unstable manifolds. Examples arise in feedback systems with state-dependent delays due to signal transmission.

Then we proceed to nonautonomous equations $y'(t) = f(t, y_t)$. For these the result in the autonomous case leads to an evolution system of continuously differentiable solution operators, under hypotheses on f which are designed for the application to equations with variable time lags, which may be unbounded. The theory covers pantograph equations and nonlinear Volterra integro-differential equations, for example. For linear nonautonomous equations we also provide a simpler evolution system with solution operators on a Banach space of continuous functions.

1. Hans-Otto Walther, “Differential equations with locally bounded delay”, J. Differential Equations, to appear.
2. Hans-Otto Walther, “Evolution systems for differential equations with variable time lags”, preprint 2011, submitted.

Hopf bifurcation for retarded FDE and for semiflows

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We discuss different approaches to the proof of Hopf bifurcation theorems, especially for retarded FDE. A geometrically oriented proof is presented that works for general semiflows, without using a special class of equations.

The attractor of slow oscillation for delayed negative feedback

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Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a smooth function with $\frac{\partial}{\partial y} f(x, y) < 0$ for all $(x, y) \in \mathbb{R}^2$, $f(0, 0) = 0$. The equation $\dot{x}(t) = f(x(t), x(t-1))$ generates a semiflow F on the space $C([-1, 0], \mathbb{R})$. Under additional conditions on f, the semiflow has a global attractor $\mathcal{A}$. We consider the subset $\mathcal{B}$ of those points in $\mathcal{A}$ which are segments of globally defined bounded solutions oscillating slowly around all equilibria. It is shown that $\mathcal{B}$ is a 2-dimensional invariant Lipschitz submanifold of the phase space which is homeomorphic to the unit disk in $\mathbb{R}^2$. The set $\mathcal{B}$ may contain stationary points, periodic, orbits, homoclinic and heteroclinic orbits.

Smooth dependence on parameters of solutions in functional differential equations with state-dependent delays

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In this talk we discuss first and second-order differentiability of the solutions with respect to parameters in several classes of functional differential equations with state-dependent delays. The differentiability is proved in a pointwise sense and also using the C-norm on the state-space, assuming that the state-dependent time lag function is piecewise strictly monotone. We also give sufficient conditions to imply Lipschitz continuity of the derivative with respect to the parameters. As an application of our result, we study a parameter estimation method using a quasilinearization technique. We define the method, show its convergence in special cases, and give numerical examples, where the unknown parameters are coefficient functions, delay functions and initial functions.