Poisson structure on the dual Hopf algebra

Viacheslav Kalnitsky skalnitsky@hotmail.com
Saint-Petersburg state university, Russia

Abstract

Let us consider a commutative associative unitary algebra A over a commutative unitary ring K and a tensor differentially closed affine subcategory of $\mathcal{M}od(A)$. The graded algebra $\oplus_i \mathrm{Csm}_i(A)$ of cosymbols of differential operators is the Hopf algebra. Thus the dual algebra $\oplus_i \mathrm{Hom}_A(\mathrm{Csm_i}(A), A)$ is commutative unitary algebra and coincides with the graded algebra of symbols of differential operators. It turned out that the Poisson bracket on it is generated by the structure of the Hopf algebra itself due to the universal differentiation $\mathrm{d} : A \to \Lambda^1(A) \cong \mathrm{Csm}_1(A)$. Namely, we can express the Poisson bracket directly

$$\{f, g\}(\theta) = \sum \left[f\left(\mathrm{d}(g(\theta_{(1)})) \cdot \theta_{(2)}\right) - g\left(\mathrm{d}(f(\theta_{(1)})) \cdot \theta_{(2)}\right)\right],$$

where $\Delta(\theta) = \sum \theta_{(1)} \otimes \theta_{(2)}$ is co-multiplication, elements of the dual algebra f, g are considered as A-homomorphisms and dot-multiplication denotes those in the Hopf algebra.

AMS Classification: 16W30.