Sub-Riemannian geometry on infinite-dimensional manifolds

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Abstract

We generalize the concept of sub-Riemannian geometry to infinite-dimensional manifolds modeled on convenient vector spaces. On a sub-Riemannian manifold M, the metric is defined only on a sub-bundle $\mathcal{H}$ of the tangent bundle TM, called the horizontal distribution. Similarly to the finite-dimensional case, we are able to split possible candidates for minimizing curves into two categories: semi-rigid curves that depend only on $\mathcal{H}$, and normal geodesics that depend both on $\mathcal{H}$ itself and on the metric on $\mathcal{H}$. In this sense, semi-rigid curves in the infinite-dimensional case generalize the notion of singular curves for finite dimensions. In particular, we study the case of regular Lie groups. As examples, we consider the group of sense-preserving diffeomorphisms Diff S^1 of the unit circle and the Virasoro-Bott group with their respective horizontal distributions chosen to be the Ehresmann connections with respect to a projection to the space of normalized univalent functions. In these cases we prove controllability and find formulas for the normal geodesics with respect to the pullback of the invariant Kählerian metric on the class of normalized univalent functions. The geodesic equations are generalizations in some sense of the Camassa-Holm, Hunter-Saxton, KdV, and other known non-linear PDE.

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