The 1-d Schrödinger operators with complex-valued distributions as potentials

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Abstract

On the Hilbert space $L^2(\mathbb{R})$ we consider the 1-d Schrödinger operators:

$$\mathrm{S}(q)u := -u'' + q(x)u, \quad q \in H^{-1}_{unif} \ \left(H^{-1}(\mathbb{R}) \subset H^{-1}_{unif} \subset H^{-1}_{loc}(\mathbb{R})\right),$$

where H^{-1}_{unif} is the negative Stepanov space ($\forall \psi \in C_0^\infty$, $\psi_a := \psi(x-a)$):

$$H^{-1}_{unif} \equiv H^{-1}_{unif}(\mathbb{R}) := \{f \in \mathfrak{D}'(\mathbb{R}) \mid \sup_{a\in\mathbb{R}} \|\psi_a f\|_{H^{-1}(\mathbb{R})} < \infty\}.$$

The operators $\mathrm{S}(q)$ can be well defined on the space $L^2(\mathbb{R})$ in different ways: as operators form-sums, as quasi-differential operators (minimal, maximal and the Friedrichs extensions), as a limit in norm resolvent sense of the sequence of operators with smooth potentials.

We prove that all definitions are equivalent and the operators $\mathrm{S}(q)$ are m-sectorial. We establish that spectra of $\mathrm{S}(q)$ belong to the parabola:

$$|\operatorname{Im}\lambda| \leq 5K\left(\operatorname{Re}\lambda + 4(2K+1)^4\right)^{3/4}, \qquad K = K(\|q\|_{H^{-1}_{unif}}) > 0.$$

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