Products of homogeneous subspaces in free Lie algebras

Let L be a free Lie algebra of finite rank over a field F and let L_n denote the degree n homogeneous component of the subspaces $[L_m, L_n]$ for all m and n were obtained by R. Stöhr and M. Vaughan-Lee (2009). Our work is concerned with subspaces of the form $[L_m, L_n, L_k] = [[L_m, L_n], L_k]$ for certain values of m, n and k. Surprisingly, in contrast to the case of a product of two homogeneous components, the dimension of such products may depend on the characteristic of the field F. For example, the dimension of $[L_2, L_2, L_1]$ over fields of characteristic 2 is different from the dimension over fields of characteristic other than 2. Our main result are formulae for the dimension of $[L_m, L_n, L_k]$. Under certain conditions on m, n and k they lead to explicit formulae that do not depend on the characteristic of F, and express the dimension of $[L_m, L_n, L_k]$ in terms of Witt's dimension function.