On Fundamental Sequence of Rademacher Subspace

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Abstract

Let E be rearrangement invariant (r.i.) space on $[0,1]$. Rademacher subspace $R(E)$ consists of all sequence $x=(x_n)$ such that $\sum_{k=1}^{\infty} x_k r_k$ converges in E and $\|x\|_{R(E)} = \|\sum_{k=1}^{\infty} x_k r_k\|_E$, $r_n(t) = sign \sin 2^n \pi t$.

Fundamental function of r.i. space E is defined by $\phi_E(t) = \|\chi_{[0,t]}\|_E$. Fundamental sequence of space E with symmetric basis $\{e_i\}_{i=1}^{\infty}$ is the following $\phi_E(n) = \left\| \sum_{i=1}^{n} e_i \right\|$.

Theorem 1. Let $E = \Lambda_\psi$ be Lorentz function space with $\phi_E = \psi$. The following relation

$$C_1 n \phi_E(2^{-n}) \le \phi_{R(E)}(n) \le C_2 n \phi_E(2^{-n}),\ 0 < C_1 \le C_2 < \infty, \quad (1)$$

implies the existence of $\alpha \in (0,1/2)$ such that $\psi(t) \ge \frac{C}{\ln^\alpha(e/t)}, C > 0$, and follows from the existence of $\alpha \in (0,1/2)$ such that $\psi(t)\ln^\alpha(e/t)$ is decreasing in some neighborhood of 0.

Theorem 2. Let $E = M_{t/\psi(t)}$ be Marcinkiewicz function space with $\phi_E = \psi$. The relation (1) implies that $\psi(t) \ge \frac{C}{\sqrt{\ln(e/t)}}$, $C > 0$, and follows from the fact that $\psi(t)\sqrt{\ln(e/t)}$ is decreasing in some neighborhood of 0.

Using the fact that $\Lambda_\psi \subseteq E \subseteq M_{t/\psi(t)}$ for any E with $\phi_E = \psi$, similar results can be obtained for arbitrary r.i. space E.

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