

Multidimensional periodic and almost-periodic spectral problems: Bethe-Sommerfeld Conjecture and integrated density of states

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Abstract

I will talk about recent progress in the study of the spectral properties of periodic and almost-periodic differential and pseudo-differential operators acting in \mathbb{R}^d , $d \geq 2$. There will be two major types of results I will discuss.

1. The Bethe-Sommerfeld Conjecture. This conjecture was originally formulated for 3-dimensional Schrödinger operators with periodic potentials and stated that the spectrum of such operators cannot have infinitely many gaps. While proved in this generality in 1985 by M.Skriganov, the conjecture was still open for other types of operators. I will describe recent results where the conjecture was established for a wide (and almost optimal) class of periodic pseudo-differential operators, including magnetic Schrödinger operators in all dimensions. These are joint results with A.Sobolev.

2. The asymptotic behaviour of the integrated density of states. I will discuss my joint result with R.Shterenberg where we have obtained a complete asymptotic expansion of the integrated density of states of Schrödinger operators for large energies. This expansion holds when the potential is either smooth periodic, or generic quasi-periodic, or belongs to a wide class of almost-periodic functions.

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