Triangle-free intersection graphs of segments have arbitrarily large chromatic number

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Abstract

We are given a finite set of planar shapes of a particular kind (for example, line segments), such that no three of the shapes pairwise intersect. We consider the problem of coloring the shapes in such a way that no two intersecting ones are of the same color. Is the number of necessary colors bounded by some universal constant depending only on the kind of shapes considered?

Paul Erdős asked this question for line segments. A. Gyárfás and J. Lehel asked it for so called L-shapes, which are shapes consisting of two perpendicular axis-aligned line segments meeting at a common endpoint. McGuiness conjectured that the answer is positive for curves every pair of which intersect in at most one point. Finally, A. D. Scott generalized these questions and conjectured that for any graph G, the class of triangle-free graphs with no induced subdivision of G has bounded chromatic number.

We show that the answer to all of the above questions is negative. To this end, we construct geometric representations of graphs encoding strategies for some online coloring problems.

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