Explicit solution of $\mathfrak{a}_1$-type Lie-Scheffers system and general Riccati equation

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Abstract

We compute the explicit solution for a general differential system $\dot{x}(t) = u_a(t)X_a + u_b(t)X_b + u_c(t)X_c$ on a manifold M, where $X_a, X_b, X_c \in \Gamma(M)$ generate the simple Lie algebra of type $\mathfrak{a}_1$. More precisely, the solution is given as a composition of flows:

$$x(t) = \exp(\Xi_c(t)X_c) \circ \exp(\Xi_b(t)X_b) \circ \exp(\Xi_a(t)X_a)(x(0)),$$

and functions $\Xi_a, \Xi_b, \Xi_c : [0, T] \to \mathbb{R}$ are given in terms of series of iterated integrals of products of u_a, u_b and u_c's. For bounded measurable u_d's the series are convergent for a small T. As a byproduct we obtain the solution of a general Riccati equation.

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