Probabilistic Methods for Partial Differential Equations

ORGANIZER: Dan Crisan (*Imperial College, London, UK)*

Monday, July 2, 17:15–19:15, Large Hall A

TALKS:

Terry Lyons (*University of Oxford, UK*), COAUTHORS: Ni Hao, Nicolas Victoir, Christian Litterer, Wonjung Lee, **The expected signature of a stochastic process. Some new PDE's and some applications**

Andrew Stuart (*University of Warwick, UK*), **Filtering the Navier-Stokes equation**

Nizar Touzi (*Ecole Polytechnique, Paris, FR*), **Viscosity solutions of fully nonlinear path-dependent PDEs**

Dan Crisan (*Imperial College, London, UK*), COAUTHORS: Francois Delarue, **Gradient bounds for solutions of semi-linear partial differential equations**

The expected signature of a stochastic process. Some new PDE's and some applications

Terry Lyons
University of Oxford, UK

How can one describe a probability measure of paths? And how should one approximate to this measure so as to capture the effect of this randomly evolving system. Markovian measures were efficiently described by Stroock and Varadhan through the Martingale problem. But there are many measures on paths that are not Markovian and a new tool, the expected signature provides a systematic ways of describing such measures in terms of their effects. Even in the Markovian context, considerable value can be extracted from computing the expected signature as it can be used to construct Cubatures on path space and when combined with a technique known as recombination, it is developing into a new and powerful approach to some numerical problems. The key is that it allows managed populations of carefully selected scenarios that effectively capture tail behaviour. Joint work with Ni Hao, Nicolas Victoir, Christian Litterer, Wonjung Lee.

Coauthors: Ni Hao, Nicolas Victoir, Christian Litterer, Wonjung Lee

Filtering the Navier-Stokes equation

Andrew Stuart
University of Warwick, UK

Filters aim to combine a dynamical model with data to update the state. In this talk we study filtering for infinite dimensional problems in which the underlying dynamical system is chaotic, so that solutions diverge, and the data is partial, involving only a finite number of projections. Thus neither the dynamics nor the observations alone can induce a contraction of the filter towards the true signal. We show, however, that the combination of the two mechanisms can lead to a contraction towards the true signal. To illustrate ideas we work with the Navier-Stokes equation and concentrate on the 3DVAR filter developed in meteorology.

Viscosity solutions of fully nonlinear path-dependent PDEs

Nizar Touzi
Ecole Polytechnique, Paris, FR

We propose a notion of viscosity solutions for path dependent fully nonlinear parabolic PDEs. One typical example is the path dependent HJB equations, which can also be viewed as viscosity solutions of second order Backward SDEs and G-martingales. The definition is based on a nonlinear optimal stopping problem, and is consistent with the notion of classical solution in the sense of Dupire's functional Itô calculus. We prove the existence, uniqueness, stability, and comparison principle for the viscosity solutions. Our approach is to use a variation of the Peron's approach to prove the comparison principle.

Gradient bounds for solutions of semi-linear partial differential equations

Dan Crisan
Imperial College, London, UK

I will discuss a recent continuation of the Kusuoka-Stroock programme of establishing smoothness properties of solutions of (possibly) degenerate partial differential equations by using probabilistic methods. In this work, we analyze a class of semi-linear parabolic partial differential equations for which the linear part is a second order differential operator of the form $V_0 + \sum_{i=1}^{N} V_i^2$, where $V_0, \ldots, V_N$ are first order differential operators that do not necessarily satisfy the Hörmander condition. Instead we assume that $V_0, \ldots, V_N$ satisfy a weaker condition, the so-called UFG condition. Specifically, we prove that the bounds of the higher order-derivatives of the solution along the vector fields coincide with those obtained in the linear case when the boundary condition is Lipschitz continuous, but that the asymptotic behavior of the derivatives may change because of the simultaneity of the nonlinearity and of the degeneracy when the boundary condition is of polynomial growth and measurable only. The methodology is also applied to partial differential equations with nonlinear terms with quadratic growth with respect to the first-order derivatives. This is joint work with Francois Delarue (University of Nice).

Coauthors: Francois Delarue