On some approximating formulas for modified BESSEL functions

Juri Rappoport jmrap@landau.ac.ru
Russian Academy of Sciences, Moscow

Abstract

LEBEDEV and YAKUBOVICH introduced certain index integral transforms with modified BESSEL functions of complex order. Therefore it is important to investigate the approximating formulas for second kind modified BESSEL functions $K_{\alpha+i\beta}(x)$ for values $\alpha = 0.0(0.25)1.0$.

We propose a new realization of the LANCZOS Tau method with minimal residue for the numerical solution of the second order differential equations with polynomial coefficients $(\alpha = 0.0)$. The approximating scheme of Tau method is expanded for systems of hypergeometric type differential equations $(\alpha = 0.25, \alpha = 0.5, \alpha = 0.75, \alpha = 1.0)$. Various vector perturbations are discussed. Our choice of the perturbation term is a shifted CHEBYSHEV polynomial with a special form of selected transition and normalization. For one equation minimality conditions for the perturbation term are found.

We elaborate power series expansions and numerical quadrature approaches for the computation of kernels of these LEBEDEV type index transforms - modified BESSEL functions of the second kind with imaginary order $K_{i\beta}(x)$ and with a certain complex order $K_{\alpha+i\beta}(x)$. The evaluation codes are constructed and tables of the modified BESSEL functions $K_{1/2+i\beta}(x)$ are published. We also give effective applications for the solution of mixed boundary value problems in wedge domains.

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