The centenary of the Riesz representation theorem: recent progress in characterization of Radon integrals

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Abstract

The poster is devoted to the problem of characterization of integrals as linear functionals. It takes the origin in the well-known result of F. Riesz (1909) on integral representation of bounded linear functionals by Riemann − Stiltjes integrals on a segment and is directly connected with the famous theorem of J. Radon (1913) on integral representation of bounded linear functionals by Lebesgue integrals on a compact in $\mathbb{R}^n$.

After works of J. Radon, M. Fréchet, and F. Hausdorff this problem has been concretized as the problem of extension of Radon's theorem from $\mathbb{R}^n$ to more general topological spaces with Radon measures.

The important stages of its solving are connected with such eminent mathematicians as S. Banach, S. Saks, S. Kakutani, P. Halmos, E. Hewitt, R. E. Edwards, D. Pollard, et al. Essential ideas and technical tools were worked out by A. D. Aleksandrov, M. N. Stone, D. H. Fremlin, et al. The modern stage of solving of the problem connected mainly with the works of H. König (1995–2008) and the authors (1997–2011).

Authors' results on characterization of general Radon integrals on an arbitrary Hausdorff space allow them to obtain the analogues of the Prokhorov theorem on weak compactness of families of Radon measures on Polish spaces for Tychonoff and Hausdorff spaces.

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