Semigroups of Operators: Theory and Applications

Organizers: Adam Bobrowski (*Lublin University of Technology, PL)*, Y. Tomilov (*TU Dresden, DE)*, Ralph Chill (*TU Dresden, DE)*

Friday, July 6, 10:45–12:45, Small Hall

Talks:

Jacek Banasiak (*University of KwaZulu-Natal, ZA & Technical University of Łódź, PL*), **Structured population models with fast migration – the case of a reducible migration matrix**

Charles Batty (*St. John's College, Oxford, UK*), **Functional calculus for semigroup generators**

Jose Gale (*University of Zaragoza, ES*), **Extension problem and fractional powers of operators**

Lutz Weis (*Universität Karlsruhe, DE*), Coauthors: J. van Neerven and M. Veraar, **Evolution equations in square function spaces**

Structured population models with fast migration – the case of a reducible migration matrix

Jacek Banasiak
University of KwaZulu-Natal, ZA & Technical University of Łódź, PL

In some recent papers the authors have studied asymptotic limits of structured models with fast migration but only in the case when the migration matrix was irreducible. This property yielded a well defined one dimensional, with respect to the structure, limit space in which the evolution could be approximated by a scalar evolution equation. In the presented paper we will extend this result to reducible migration matrices. In particular, we will show that the limit space becomes multidimensional, determine the spectral projections onto this space, provide their interpretation and show that the approximated evolution there is described by a system of coupled equations.

References

[Ar] Arino, O., Sánchez, E., Bravo de la Parra, R. and Auger, P., A singular perturbation in an age-structured population model, *SIAM Journal on Applied Mathematics*, **60**(2), 1999, 408–436.
[bgs] Banasiak, J., Goswami, A. and Shindin, Aggregation in age and space structured population models: an asymptotic analysis approach *J. Evol. Equ.* **11** (2011), 121–154.
[ToLi] Lisi, M., Totaro, S., The Chapman-Enskog procedure for an age-structured population model: initial, boundary and corner layer corrections, *Math. Biosci.*, **196**(2), 2005, 153–186.

Functional calculus for semigroup generators

Charles Batty
St. John's College, Oxford, UK

Bounded H^∞-calculus for sectorial operators (generators of holomorphic semigroups) is well established, and there is now a parallel theory for strip-type operators (generators of groups). There is a corresponding notion for half-plane operators (generators of semigroups). The talk will describe the theory in that case, when the results are less powerful but there are some connections with integral resolvent conditions.

Extension problem and fractional powers of operators

Jose Gale
University of Zaragoza, ES

Recent theorems of Caffarelli-Silvestre and Stinga-Torrea provide solutions to certain extension problems for differential operators, which include the examples of the Laplacian and the harmonic oscillator in $\mathbb{R}^n$. The solutions can be expressed in terms of fractional powers of the differential operators. Conversely, such fractional powers can be recovered from the solutions and, in this way, those powers appear as a kind of Poisson-to-Neumann correspondence. In the talk, a fairly general extension of the above theorems will be given, in the setting of operator semigroups and their generators. The result applies in particular to all infinitesimal generators of bounded C_0-semigroups and operators with pure imaginary symbols.

Evolution equations in square function spaces

Lutz Weis
Universität Karlsruhe, DE

We consider deterministic and stochastic evolution equations

$$dY(t) \;=\; [A(t)\,Y(t) + F(Y(t))]\;dt \;+\; B(Y(t))\;dW(t) \qquad (1)$$

of semilinear and fully nonlinear type on UMD Banach spaces X with square function norms. Typically, for $X = L^p(U,\mu)$ and functions $f\colon I \to X$, these norms are of the form

$$\|f\|_{\gamma(X)} \;=\; \left\| \int_I \left(|f(t)|^2 \; dt \right)^{\frac{1}{2}} \right\|_{L^p} .$$

They are very prominent in harmonic analysis, spectral theory, and in stochastic calculus for X-valued functions. Since these are precisely the tools for finding mild solutions of (1), our framework allows for natural results under reasonably general conditions, also in Banach spaces that are not function spaces. We illustrate our results with some parabolic partial differential equations and further explain the essential properties of square function norms, which make them such a convenient tool for the study of evolution equations of type (1).

Coauthors: J. van Neerven and M. Veraar