Study of the equilibrium of the pendant drop.

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Let $\Im_H$ be a class of axisymmetrical surfaces generated by the curves whose natural parameterizations are twice differentiable in a generalized sense. We suppose that symmetry's axis of the surface from the class $\Im_H$ is orthogonal to the plane P. On the class $\Im_H$ we consider the functional

$$F = F(S) := \sigma \cdot A + \sigma \cdot l_p \cdot \Xi - \beta \cdot \sigma \cdot \int\limits_{S^\bullet} dS + \lambda \cdot \sigma \cdot V +$$

$$+ \iiint\limits_W \Gamma \cdot \rho \cdot dV + \mu \cdot \sigma \cdot H(\, S\,),$$

$$\Xi := 2\pi \int\limits_0^L \left\{ -\sqrt{1-\dot{y}^2} \int\limits_0^{\dot{y}} \left(\arcsin \sigma + \sigma \cdot \sqrt{1-\dot{\sigma}^2} - \frac{\pi}{2} \right) \times \right.$$

$$\left. \times \left(\, 1 - \dot{\sigma}^2 \right)^{-\frac{3}{2}} d\sigma + E_0 \sqrt{1-\dot{y}^2} \right\},$$

$$H(S) := \|H\|^2_{2,S} = \int\limits_S H^2 dS < \infty.$$

Here the symbol A denotes the area of the surface $S, S^\bullet$ - the disc inside of the line L of the intersection between S and P, W - the domain comprised between S and P, V-its volume and H denotes the mean curvature H of an admissible surface. We suppose the function Γ to be continuous and non negative and the numbers $E_0, l_p, \beta, \lambda, \mu, \rho, \sigma$- non negative.

Theorem. There exists a surface $S_e \in \Im_H$ delivering a minimal value to the functional F over the class $\Im_H$ whose mean and gauss curvatures H, K satisfy the following equation

$$\mu \cdot \Delta_S H + 2\mu \cdot H \cdot \left(H^2 - K \right) + 2 \cdot H + l_p K = \lambda + \frac{1}{\sigma} \cdot \Gamma \cdot \rho$$

Here Δ_S denotes the Laplace –Beltrami operator of the surface$S$. The problem of this type arises in the study of the equilibrium state of the pending drop when intermediate layer and its flexural rigidity are taken into account. In our study we obtain also new condition for the contact angle between S and P.