On a category of highest weight representations of a semiclassical Lie group $D_{n-1/2}$

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Abstract

The semiclassical Lie groups $G_{n-1/2}$ (where $G_n = A_n$ or B_n or C_n or D_n are the classical Cartan series of simple Lie groups) where introduced by author [1] to separate multiple points of spectrum in reductions of type $G_n \downarrow G_{n-1}$. Let T_m be a finite dimensional irreducible representation of a complex simple Lie group G_n, m is a highest weight of T_m, $m \in P_n$. The branching rules for all Cartan series may be written in an uniform way:

$$T_m|_{G_{n-1}} = \bigoplus_{\lambda \to m} \left(\bigoplus_{t \to \lambda} T_t \right), \quad m \in P_n, \quad t \in P_{n-1},$$

where the summation ranges over all weights λ satisfying certain subordination conditions depending on the series. It turns out that the space $\bigoplus\limits_{t \to \lambda} T_t$ may be endowed with a structure of a module $L(\lambda)$ over some Lie group intermediate between G_{n-1} and G_n. We denote it by $G_{n-1/2}$. Note that the reductions $G_n \downarrow G_{n-1/2}$ and $G_{n-1/2} \downarrow G_{n-1}$ are multiplicity–free.

Let L be a category of $D_{n-1/2}$–factors of filtrations that separate the isomorphic components of $T_m|_{G_{n-1}}$. The factors in question are cyclic and generated by a single vector of a highest weight. We call L the category of highest weight representations and investigate the characters and dimensions of $D_{n-1/2}$–modules $L(\lambda)$.

Theorem. [2] *Let $g \in D_{n-1/2}$, $\{z_1, ..., z_{n-1}, 1, 1, z_{n-1}^{-1}, ..., z_1^{-1}\}$ is a set of eigenvalues of g. Then the character of a representation $L(\lambda_1, ..., \lambda_{n-1}, \lambda_n, \lambda_n)$ with a highest weight $(\lambda_1, ..., \lambda_{n-1}, \lambda_n, \lambda_n) \in P_{n+1}$ of Lie group $D_{n-1/2}$ can be calculated by the next formula:*

$$\mathrm{ch}\left[L(\lambda_1, ..., \lambda_{n-1}, \lambda_n, \lambda_n)\right](g) = \frac{D_1(\lambda) + D_2(\lambda)}{D_1(0)},$$

where

$$D_1(\lambda) = \det \begin{pmatrix} S(z_1^{\tau_1}) & \ldots S(z_1^{\tau_{n-2}}) & C(z_1^{\tau_{n-1}}) \cdot S(z_1^{1/2}) & S(z_1^{\tau_n}) \\ \multicolumn{4}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots} \\ S(z_{n-1}^{\tau_1}) & \ldots S(z_{n-1}^{\tau_{n-2}}) & C(z_{n-1}^{\tau_{n-1}}) \cdot S(z_{n-1}^{1/2}) & S(z_{n-1}^{\tau_n}) \\ 1 & \ldots \quad 1 & 0 & 1 \end{pmatrix},$$

and we use designations $S(z^\tau) = z^\tau - z^{-\tau}$, $C(z^\tau) = z^\tau + z^{-\tau}$; $\tau_i = \lambda_i + n - 1/2$ *for* $i = 1, 2, ..., n-2$; $\tau_{n-1} = \lambda_{n-1}$; $\tau_n = \lambda_n + n - 1/2$. *The determinant* $D_2(\lambda)$ *can be obtained from* $D_1(\lambda)$ *by substitution* $S(z^\tau)$ *on* $C(z^\tau)$ *and vice versa.*

We compare the discovered formulas for characters and dimensions of the highest weight representations of semiclassical Lie groups with well-known Hermann Weyl analogous formulas.

[1]. Shtepin V.V. *The intermediate Lie algebra* $\mathfrak{d}_{n-1/2}$, *the weight scheme and finite–dimensional representations*. Izvestiya: Mathematics **68**:2, 2004, p. 375–404.

[2]. Shtepin V.V., Konashenkov D.L. *The characters and dimensions of highest weight representations of intermediate Lie algebra* $\mathfrak{d}_{n-1/2}$. Izvestiya: Mathematics **76** (to appear).

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