On the relative equilibrium configurations in the planar 6-body problem

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Abstract

A central configuration of six point-masses (m_i, q_i) ($q_i \in R^2$, $m_i \in R^+$, $i = 1, \ldots, 6$) located at the edges of a segment of lenght $2q$ with the rest four point-masses at the vertices of a square with the side $q'\sqrt{2}$ has been presented. The segment is contained into a diagonal of the square and both have the same center of symmetry. Necessary and sufficient conditions imposed on the masses (the masses of the bodies located at the vertices of the square need not be equal) and radii for such a central configuration to exist have been given. By a *central configuration* in a barycentric frame of reference of N material points (m_i, q_i), $m_i \in R^+$, $q_i = (x_i, y_i) \in R^2$, $i = 1, \ldots, N$ we understand a configuration such that the total Newtonian acceleration on every body is proportional with the position vector of this body with respect to the center of mass of the configuration, i.e., $\ddot{q}_i = \sigma \cdot q_i$, for some $\sigma \neq 0$ (see, e.g., [0]). This system also may be written as

$$\frac{\partial U}{\partial x_i} = \sigma \frac{\partial I}{\partial x_i}, \quad \frac{\partial U}{\partial y_i} = \sigma \frac{\partial I}{\partial y_i} \quad 1 \leq i \leq N,$$

where $U = \sum_{i<j} m_i m_j / (x_{ij}^2 + y_{ij}^2)^{1/2}$ and $I = \sum_{i=1}^{N} m_i (x_i^2 + y_i^2)$ are respectively the Newtonian potential and the momentum of inertia. Central configurations are interesting because they allow to obtain explicit homographic solutions of the N-body problem. Central configurations also appear as a key point when we study the topology of the fibers in the phase space with fixed energy and angular momentum i.e., the fibers of the energy-momentum mapping (see [0]). The number of similarity classes of planar central configurations in the N-body problem for an arbitrary given set of positive masses has been established only for $N = 3$. In this case L.Euler has found a collinear relative equilibrium, and J.L.Lagrange has found central configurations as two equilateral triangles (see [0] and the bibliography therein). In the 90-ies B.Elmabsout [0] and E.A.Grebenicov [0] have proved that besides of the class of gravitational models in the inertial barycentric system (the so-called gravitational model of Lagrange-Wintner), there exists a new class of gravitational models, i.e., the class of gravitational models in non inertial frames (the so-called gravitational model of Grebenicov-Elmabsout, we denote it by $GE(m_0, N)$). They have proved that there exists a relative equilibrium configuration in the $(N + 1)$- body problem with N

bodies with equal masses located at the vertices of a regular N-gon, and with a body of non-zero mass situated in the center of the polygon. In 1991 B.Elmabsout has stated necessary and sufficient conditions for the existence of a relative equilibrium configuration of the $(N+1)$-body problem for the Grebenicov-Elmabsout models when N material particles are located at the vertices of p regular n-gons centered at a given point-mass m_0, with the bodies on the same n-gon having equal masses, and therefore $N = p \cdot n$ [0]. We will denote this class of models by $GE(m_0, p, n)$. During the last two decades in the N-body problem a series of papers on central configurations of type $GE(m_0, p, n)$ with $p = 1, 2, 3$ and $n = 2, 3, 4, 5$ arose (E.A.Grebenicov, N.I.Zemtsova, A.Siluszyk, E.V.Ihsanov, D.Kozak-Skoworodkin). The problem of existence, finiteness and the evaluation of the number of central configurations in an asymmetric N-body problem for $N = 5, 7$ has been stated by A.Siluszyk [0, 0].

We are concerned with the existence problems in two cases: namely $(0, 2, 4)$ and $(0, 4, 2)$. In the first case for $0 < q < q'$, we assume, that the "interior" two-points have equal masses, say m, whilst the "exterior" four bodies have the masses m_3, m_4, m_5 and m_6 enumerated clockwise [0]. In the second case, for $0 < q' < q$, we suggest that four bodies having the masses m_3, m_4, m_5 and m_6 are "interior", whereas the rest two-points with mass m are "exterior".

Theorem *Given the real numbers $m > 0$ and $q > q' > 0$ a central configurations of the type $(0, 4, 2)$ exist if and only if the masses m_3, m_4, m_5 and m_6 satisfy the following two conditions:*

(1) $m_3 = m_5$ *and* $m_4 = m_6$,

(2) $\alpha_1 m + \alpha_2 m_3 + \alpha_3 m_4 = 0$,

where the coefficients $\alpha_1, \alpha_2, \alpha_3$ depend only on radii q, q' and are defined by the formulas:

$$\begin{cases} \alpha_1 = \frac{1}{4q^3} - \frac{2}{(q^2+q'^2)^{\frac{3}{2}}}, \\ \alpha_2 = \frac{2\sqrt{2}-1}{4q'^3} + \frac{\left(-\frac{1}{4q'^3} + \frac{2}{(q^2+q'^2)^{\frac{3}{2}}}\right)\left(\frac{1}{4q^3} - \frac{4q}{q^2-q'^2}\right)}{\frac{1}{4q^3} - \frac{2}{(q^2+q'^2)^{\frac{3}{2}}}}, \\ \alpha_3 = \frac{1-2\sqrt{2}}{4q'^3} + \frac{\left(\frac{1}{4q^3} + \frac{4q}{q^2-q'^2}\right)\left(-\frac{1}{\sqrt{2}q'^3} + \frac{2(q^2+q'^2)}{q(q^2-q'^2)^2}\right)}{\frac{1}{4q^3} - \frac{2}{(q^2+q'^2)^{\frac{3}{2}}}} \end{cases} \qquad (1)$$

A computer-assisted proof of this theorem makes use of CAS Mathematica.

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