Combinatorics of asymptotic representation theory

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Abstract

The representation theory of the symmetric groups S_n since its very beginning has been intimately related to combinatorics: combinatorial objects such as Young tableaux and combinatorial algorithms such as Littlewood-Richardson rule. Unfortunately, in the limit as n tends to infinity, the structure of these combinatorial objects and algorithms becomes very complicated and cumbersome and it is hard to extract from them some meaningful answers to asymptotic questions. In order to overcome these difficulties, since 1990s a kind of dual combinatorics of the representation theory of the symmetric groups was investigated. This dual combinatorics turned out to be highly successfull for solving asymptotic problems: one of its applications is a connection (discovered by Biane) between representations of the symmetric groups, random matrices and Voiculescu's free probability theory. During the lecture I will concentrate on one of the highlights of this new combinatorics: Kerov polynomials which express characters in terms of free cumulants (quantities which originate from free probability and random matrix theory). It is known, for example, that the coefficients of Kerov polynomials are equal to a generalization of Hurwitz numbers, but several mysteries concerning their structure remain.