Frame mutlipliers - invertibility and representation for the inverse

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Abstract

Frames are sequences $(\phi_n)_{n=1}^{\infty}$ in a Hilbert spaces $\mathcal{H}$ so that there exist positive constants A and B with

$$A\|h\| \leq \sum_{n=1}^{\infty} |\langle h, \phi_n \rangle|^2 \leq B\|h\|, \forall h \in \mathcal{H}.$$

Frames generalize orthonormal bases and find many applications in signal and image processing. Multipliers are operators in the form

$$M_{(m_n),(\phi_n),(\psi_n)}h = \sum_{n=1}^{\infty} m_n \langle h, \psi_n \rangle \phi_n, \ h \in \mathcal{H},$$

where $(\phi_n)_{n=1}^{\infty}$ and $(\psi_n)_{n=1}^{\infty}$ are sequences with elements from a Hilbert space $\mathcal{H}$ and $(m_n)_{n=1}^{\infty}$ is a complex scalar sequence. The interest to multipliers comes from life applications, for example psychoacoustics and measurement of acoustical systems, where Gabor multipliers play an important role. Since multipliers are important for applications in acoustical signal processing, it is interesting to determine their inverses. On the other hand, it is always interesting from a theoretical point of view to investigate the inverses of certain operators.

Here we consider the question for invertibility of multipliers when one of the involved sequences is a frame. We give sufficient conditions for invertibility and representation for the inverse operator via series. The bounds used in the sufficient conditions are sharp. When one of the involved sequences is a Riesz basis (i.e., a frame which is at the same time a Schauder basis), we present necessary and sufficient conditions for invertibility.

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