

The generalized Meixner-Pollaczek polynomials

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Abstract

In this work we consider the generalized Meixner-Pollaczek (GMP) polynomials $P_n^\lambda(x; \theta, \psi)$ of variable $x \in \mathbb{R}$ and parameters $\lambda > 0$, $\theta \in (0, \pi)$, $\psi \in \mathbb{R}$ via the generating function

$$G^\lambda(x; \theta, \psi; z) = \frac{1}{(1 - ze^{i\theta})^{\lambda-ix}(1 - ze^{i\psi})^{\lambda+ix}} = \sum_{n=0}^{\infty} P_n^\lambda(x; \theta, \psi) z^n, \quad |z| < 1.$$

We find the three-term recurrence relation, the explicit formula, the hypergeometric representation, the difference equation and we prove the orthogonality relation for (GMP) polynomials $P_n^\lambda(x; \theta, \psi)$.

Moreover, we study the GMP and the special case of $P_n^\lambda(x; \theta, \psi)$ corresponding to the choice $\psi = \pi + \theta$ and $\psi = \pi - \theta$, which lead to some interesting families of polynomials. The limiting case ($\lambda \rightarrow 0$) of the sequences of polynomials $P_n^\lambda(x; \theta, \pi + \theta)$ is obtained, and is shown to be an orthogonal sequence in the strip,

$S = \{z \in \mathbb{C} : |\operatorname{Im}(z)| < 1\}$.

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