Optimal control and applications to aerospace problems

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Shooting method

Orbit transfer

Three-body problem

What is control theory?

Objective

Steer a system from an initial configuration to a final configuration.

Optimal control

One tries moreover to minimize a given criterion.

Stabilization

A trajectory being planned, one tries to stabilize it in order to make it robust, insensitive to perturbations.

Observability

Reconstruct the full state of the system from partial data.







Application domains of control theory:

Mechanics

Vehicles (guidance, dampers, ABS, ESP, ...), Aeronautics, aerospace (shuttle, satellites), robotics







Electricity, electronics

RLC circuits, thermostats, regulation, refrigeration, computers, internet and telecommunications in general, photography and digital video



Chemistry

Chemical kinetics, engineering process, petroleum, distillation, petrochemical industry





Control theory and applications







Predator-prey systems, bioreactors, epidemiology, medicine (peacemakers, laser surgery)





Economics

Gain optimization, control of financial flux, Market prevision





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Orbit transfer







Here we focus on applications of control theory to problems of aerospace.







Optimal control and applications to aerospace problems

Orbit transfer

The orbit transfer problem with low thrust

Controlled Kepler equation

$$\ddot{q} = -q\frac{\mu}{r^3} + \frac{F}{m}$$

 $q \in \mathbb{R}^3$: position, r = |q|, *F*: thrust, *m* mass:

 $\dot{m} = -\beta |F|$

Maximal thrust constraint

$$|F| = (u_1^2 + u_2^2 + u_3^2)^{1/2} \leqslant F_{\max} \simeq 0.1N$$

Orbit transfer

from an initial excentric inclinated orbit to a *geostationary* orbit.

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Shooting method

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Three-body problem

The orbit transfer problem with low thrust

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Modelization in terms of an optimal control problem

State:
$$x(t) = \begin{pmatrix} q(t) \\ \dot{q}(t) \end{pmatrix}$$

Control: u(t) = F(t)

Optimal control problem

$$\dot{x}(t) = f(x(t), u(t)), \qquad x(t) \in \mathbb{R}^n, \quad u(t) \in \Omega \subset \mathbb{R}^m,$$
$$x(0) = x_0, \quad x(T) = x_1,$$
$$\min C(T, u), \quad \text{where } C(T, u) = \int_0^T f^0(x(t), u(t)) dt$$

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Pontryagin Maximum Principle

Optimal control problem

$$\dot{x}(t) = f(x(t), u(t)), \ x(0) = x_0 \in \mathbb{R}^n, \quad u(t) \in \Omega \subset \mathbb{R}^m,$$

$$x(T) = x_1, \text{ min } C(T, u), \text{ where } C(T, u) = \int_0^t f^0(x(t), u(t)) dt.$$

Pontryagin Maximum Principle

Every minimizing trajectory $x(\cdot)$ is the projection of an *extremal* $(x(\cdot), p(\cdot), p^0, u(\cdot))$ solution of

$$\dot{x} = \frac{\partial H}{\partial p}, \ \dot{p} = -\frac{\partial H}{\partial x}, \qquad H(x, p, p^0, u) = \max_{v \in \Omega} H(x, p, p^0, v),$$

where $H(x, p, p^0, u) = \langle p, f(x, u) \rangle + p^0 f^0(x, u)$.

An extremal is said *normal* whenever $p^0 \neq 0$, and *abnormal* whenever $p^0 = 0$.

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Shooting method

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Pontryagin Maximum Principle

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Pontryagin Maximum Principle

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$$u(t) = u(x(t), p(t))$$
(locally, e.g. under the strict Legendre

assumption: $\frac{\partial^2 H}{\partial u^2}(x, p, u)$ negative definite)

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Pontryagin Maximum Principle

$$H(x, \rho, \rho^0, u) = \langle \rho, f(x, u) \rangle + \rho^0 f^0(x, u).$$

Pontryagin Maximum Principle

Every minimizing trajectory $x(\cdot)$ is the projection of an *extremal* $(x(\cdot), p(\cdot), p^0, u(\cdot))$ solution of

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Shooting method:

Extremals (x, p) are solutions of

$$\dot{x} = \frac{\partial H}{\partial p}(x, p), \quad x(0) = x_0, \qquad (x(T) = x_1),$$
$$\dot{p} = -\frac{\partial H}{\partial x}(x, p), \ p(0) = p_0,$$

where the optimal control maximizes the Hamiltonian.

Exponential mapping

$$\exp_{x_0}(t, p_0) = x(t, x_0, p_0),$$

(extremal flow)

 \longrightarrow Shooting method: determine p_0 s.t. $\exp_{x_0}(t, p_0) = x_1$.

There exist other numerical approaches to solve optimal control problems:

- direct methods: discretize the whole problem ⇒ finite-dimensional nonlinear optimization problem with constraints.
- Hamilton-Jacobi methods.

The shooting method is called an indirect method.

In the present aerospace applications, the use of shooting methods is priviledged in general because of their very good numerical accuracy. BUT: difficult to make converge... (Newton method)

To improve their performances and widen their domain of applicability, optimal control tools must be combined with other techniques:

- geometric tools ⇒ geometric optimal control
- continuation or homotopy methods
- dynamical systems theory

Introduction	Shooting method	Orbit transfer	Three-body problem
Orbit transfer	. minimal time		

Maximum Principle \Rightarrow the extremals (*x*, *p*) are solutions of

$$\dot{x} = \frac{\partial H}{\partial \rho}, \ x(0) = x_0, \ x(T) = x_1, \quad \dot{\rho} = -\frac{\partial H}{\partial x}, \ \rho(0) = \rho_0,$$

with an optimal control saturating the constraint: $||u(t)|| = F_{max}$.

 \longrightarrow Shooting method: determine p_0 s.t. $x(T) = x_1$,

combined with a homotopy on $F_{max} \mapsto p_0(F_{max})$

Heuristic on *t_f*:

$$t_f(F_{max}) \cdot F_{max} \simeq \text{cste.}$$

(the optimal trajectories are "straight lines", Bonnard-Caillau 2009)

F_{max}	t_f	Exécution	F_{max}	t_f	Exécution
60	14.800	1	1.4	606.13	33
24	34.716	5	1	853.31	44
12	70.249	3	0.7	1214.5	64
9	93.272	7	0.5	1699.4	234
6	141.22	6	0.3	2870.2	223
3	285.77	22	0.2	4265.7	226
2	425.61	22			

(Caillau, Gergaud, Haberkorn, Martinon, Noailles, ...)

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Main tool used: continuation (homotopy) method

 \rightarrow continuity of the optimal solution with respect to a parameter λ

Theoretical framework (sensitivity analysis):

$$\exp_{x_0,\lambda}(T,p_0(\lambda))=x_1$$

Ongoing work with EADS Astrium:

Minimal consumption transfer for launchers Ariane V and next Ariane VI (third atmospheric phase, strong thrust)

Objective: automatic and instantaneous software.

• continuation on the curvature of the Earth (flat Earth -> round Earth)

M. Cerf, T. Haberkorn, E. Trélat, Continuation from a flat to a round Earth model in the coplanar orbit transfer problem, Optimal Appl. Cont. Methods (2012).

● eclipse constraints → state constraints, hybrid systems

T. Haberkorn, E. Trélat, Convergence results for smooth regularizations of hybrid nonlinear optimic control problems, SIAM J. Control Optim. (2011).

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Optimal control

A challenge (urgent!!)

Collecting space debris:

- 22000 debris of more than 10 cm (cataloged)
- 500000 debris between 1 and 10 cm (not cataloged)
- millions of smaller debris

In low orbit

→ difficult mathematical problems combining optimal control, continuous / discrete / combinatorial optimization (PhD of Max Cerf, to be defended in 2012)

Ongoing studies, CNES, EADS, NASA

Shooting method

Orbit transfer

Three-body problem

Optimal control

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Around the geostationary orbit

 \rightarrow difficult mathematical problems combining optimal control, continuous / discrete / combinatorial optimization (PhD of Max Cerf, to be defended in 2012)

Ongoing studies, CNES, EADS, NASA

Optimal control

A challenge (urgent!!)

Collecting space debris:

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- millions of smaller debris

The space garbage collectors

 \rightarrow difficult mathematical problems combining optimal control, continuous / discrete / combinatorial optimization (PhD of Max Cerf, to be defended in 2012)

Ongoing studies, CNES, EADS, NASA

Shooting method

Orbit transfer

Three-body problem

The circular restricted three-body problem

Dynamics of a body with negligible mass in the gravitational field of two masses m_1 and m_2 (primaries) having circular orbits:

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Gomez, Jorba, Llibre, Masdemont, Simo...

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Lagrange points

Jacobi integral $J = 2\Phi - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \rightarrow 5$ -dimensional energy manifold

Five equilibrium points:

- 3 collinear equilibrium points: *L*₁, *L*₂, *L*₃ (unstable);
- 2 equilateral equilibrium points: L_4 , L_5 (stable).

(see Szebehely 1967)

Orbit transfer

Lagrange points in the Earth-Sun system

From Moser's theorem:

*L*₁, *L*₂, *L*₃: unstable.
 *L*₄, *L*₅: stable.

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Lagrange points in the Earth-Moon system

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Examples of objects near Lagrange points

Points L4 and L5 (stable) in the Sun-Jupiter system: trojan asteroids

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Examples of objects near Lagrange points

Sun-Earth system:

Point L1: SOHO

Point L2: JWST

Point L3: planet X...

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Periodic orbits

From a Lyapunov-Poincaré theorem, there exist:

- a 2-parameter family of periodic orbits around L₁, L₂, L₃
- a 3-parameter family of periodic orbits around L₄, L₅

Among them:

- planar orbits called Lyapunov orbits;
- 3D orbits diffeomorphic to circles called halo orbits;
- other 3D orbits with more complicated shape called Lissajous orbits.

(see Richardson 1980, Gomez Masdemont Simo 1998)

Examples of the use of halo orbits:

Orbit of SOHO around L1

(requires control by stabilization)

Orbit of the probe Genesis (2001-2004)

Invariant manifolds

Invariant manifolds (stable and unstable) of periodic orbits:

4-dimensional tubes $(S^3 \times \mathbb{R})$ inside the 5-dimensional energy manifold. (they play the role of separatrices)

-> invariant "tubes", kinds of "gravity currents" \Rightarrow low-cost trajectories

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Invariant manifolds

Invariant manifolds (stable and unstable) of periodic orbits:

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-> invariant "tubes", kinds of "gravity currents" \Rightarrow low-cost trajectories

Cartography \Rightarrow design of low-cost interplanetary missions

Meanwhile...

Back to the Moon

 \Rightarrow lunar station: intermediate point for interplanetary missions

Challenge: design low-cost trajectories to the Moon and flying over all the surface of the Moon.

Mathematics used: dynamical systems theory, differential geometry, ergodic theory, control, scientific computing, optimization

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Eight Lissajous orbits

(PhD thesis of G. Archambeau)

Periodic orbits around L_1 et L_2 (Earth-Moon system) having the shape of an eight:

 \Rightarrow Eight-shaped invariant manifolds:

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Invariant manifolds of Eight Lissajous orbits

We observe numerically that they enjoy two nice properties:

1) Stability in long time of invariant manifolds

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Invariant manifolds of Eight Lissajous orbits

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We observe numerically that they enjoy two nice properties:

2) Flying over almost all the surface of the Moon

Invariant manifolds of an eight-shaped orbit around the Moon:

- oscillations around the Moon
- global stability in long time
- minimal distance to the Moon: 1500 km.
- G. Archambeau, P. Augros, E.T., Eight Lissajous orbits in the Earth-Moon system, MathS in Action (2011).

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Perspectives

Partnership between EADS Astrium Space Transportation (les Mureaux, France) and FSMP (Fondation Sciences Mathématiques de Paris):

- \rightarrow starting next october 2012
- \rightarrow scientific collaboration with PhD's, postdocs
 - Planning low-cost missions to the Moon or interplanetary one, using the gravity corridors and other gravitational properties

mixed optimization:

- interplanetary missions: compromise between low cost and long transfer time; gravitational effects (swing-by)
- optimal conception of space vehicles
- collecting space debris
- optimal placement problems (vehicle design, sensors)
- Inverse problems: reconstructing a thermic, acoustic, electromagnetic environment (coupling ODE's / PDE's)
- Robustness problems

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Invariant manifolds of eight-shaped Lissajous orbits

 $\Phi(\cdot, t)$: transition matrix along a reference trajectory $x(\cdot)$ $\Delta > 0$.

Local Lyapunov exponent

$$\lambda(t, \Delta) = \frac{1}{\Delta} \ln \left(\text{maximal eigenvalue of } \sqrt{\Phi(t + \Delta, t) \Phi^{T}(t + \Delta, t)} \right)$$

Simulations with $\Delta = 1$ day.

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Shooting method

LLE of an eight-shaped Lissajous orbit:

LLE of an invariant manifold of an eight-shaped Lissajous orbit:

LLE of an halo orbit:

LLE of an invariant manifold of an halo orbit:

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