Generalized Hake's property for a Kurzweil-Henstock Type Integral

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Abstract

There are many areas in harmonic analysis which require non-absolutely convergent integration processes more powerful than the Lebesgue integration. In particular Denjoy-Perron and Kurzweil-Henstock type integrals, defined with respect to various derivation bases, are used to solve the problem of recovering, by generalized Fourier formulae, the coefficients of orthogonal series.

H. Hake proved a theorem stating that, in contrast to the Lebesque integral, the Denjoy-Perron integral on a compact interval is equivalent to the improper Denjoy-Perron integral, i.e., Denjoy-Perron integrability of a function f on $[a, b]$ is equivalent to Denjoy-Perron integrability on $[a, c]$ with $a < c < b$ together with the existence of the limit $\lim_{c \to b} \int_a^c f$. As Denjoy-Perron integral on the real line is known to be equivalent to the Kurzweil-Henstock integral, the same Hake's property is true for the last integral. The general idea of computing the improper integral as a limit of integral over increasing families $\{A_\alpha\}$ of sets can be realized in the multidimensional case in several different ways depending on the type of integral and on what family $\{A_\alpha\}$ is chosen to generalize the compact intervals of the one-dimensional construction. This gives rise to various types of Hake property. Some versions of it for certain Kurzweil-Henstock type integrals in $\mathbb{R}^n$ was studied by several authors and we discuss those versions.

We consider also a Kurzweil-Henstock type integral with respect to an abstract derivation basis on a Hausdorff topological space with an outer regular Borel measure on it. We investigate what assumptions on basis guarantee a generalized Hake property for this integral.

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