Vizualization of attractors for model of weak concentrated aqueous polymer solutions

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Abstract

There is an approach to the problem of attractors in fluid mechanics suggested by G. Sell as well as by M.I.Vishik and his pupils. It is based on the notions of a trajectory space and a trajectory attractor. But this approach requires that the trajectory space should be translation invariant. Usually trajectory spaces are constructed on the basis of a priori estimates of solutions. But all the a priori estimates established for the models of non-Newtonian fluid mechanics generate noninvariant trajectory spaces. For this reason a new construction of trajectory and global attractors was suggested by V.G. Zvyagin and D.A. Vorotnikov. This approach has been modified in order to introduce a trajectory space suitable for vizualization of attractors of non-Newtonian models. This approach allows to obtain the approximate calculation of attractors or their characteristics in the case of specific flows. In this this talk this task is implemented for the Poiseuille flow as a visualisation of its attractors. In the most general sense the visualisation is a way of a visual graphic representation.

System of equations describing the motion of weak concentrated aqueous polymer solutions may be written as follows:

$$\frac{\partial v}{\partial t} - \nu\Delta v + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \varkappa \frac{\partial \Delta v}{\partial t} + \nabla p = f, \tag{1}$$

$$div\, v = \sum_{i=1}^{n} \frac{\partial v_i}{\partial x_i} = 0. \tag{2}$$

We shall consider this system in a bounded domain $\overline{\Omega} \subset \mathbb{R}^2, \overline{\Omega} = [-R;R] \times [-1;1]$ on a time interval $\mathbb{R}_+ \equiv [0,+\infty)$ with boundary conditions

$$v_1\big|_{y=\pm 1} = v_2\big|_{y=\pm 1} = 0. \tag{3}$$

Travelling wave perturbations of stable Poiseuille flow for model of weak concentrated aqueous polymer solutions has been considerated. The travelling wave perturbations have the form

$$\tilde{v}_1(x,y,t) = Re\left(\mathbf{u}(y)e^{i\alpha(x-ct)}\right),$$

$$\tilde{v}_2(x,y,t) = Re\left(\mathbf{w}(y)e^{i\alpha(x-ct)}\right),$$

$$\tilde{p}(x,y,t) = Re\left(\mathbf{p}(y)e^{i\alpha(x-ct)}\right),$$

where the complex functions $\mathbf{u}(y), \mathbf{w}(y), \mathbf{p}(y)$ depend neither on the coordinate x nor on time. The parameters $\alpha > 0$ and $c \in \mathbb{C}$ characterise the wave. Namely, α describes its amplitude, $Re\, c$ describes its velocity and $Im\, c$ describes the rate of amplitude growth.

The following equation for small perturbations has been obtained:

$$(\nu - ic\alpha\varkappa)\,\mathbf{w}^{\mathrm{IV}}(y) + \left(2ic\alpha^3\varkappa - 2\nu\alpha^2 - i\alpha(1-y^2) + ic\alpha\right)\mathbf{w}''(y) + \\ + \left(\nu\alpha^4 - ic\alpha^5\varkappa + i\alpha^3(1-y^2) - ic\alpha^3 - 2i\alpha\right)\mathbf{w}(y) = 0.$$

Similar equation for the Navier-Stokes system usually is called Orr-Sommerfeld equation.

The perturbations with $Im\, c < 0$ are decaying, that is the zero perturbation is their limit regime; the perturbations with $Im\, c > 0$ are increasing, and the linearised model becomes inadequate for them after a while; the perturbations with $Im\, c = 0$ are periodic with respect to time t. Various conditions for periodicity ($Im\, c = 0$), damping ($Im\, c < 0$) and crushing (infinite increase) ($Im\, c < 0$) of the perturbations have been obtained.

Using these results a visualisation of attractors of the considered problem has been obtained as follows:

Let us introduce the space $E = C(\overline{\Omega}, \mathbb{R}^3)$ consists of continuous vector functions and it is equipped with the norm

$$\|u\|_E = \max_{(x,y)\in\overline{\Omega}} |u(x,y)|.$$

We choose a number $R_0 > 0$ and define the trajectory space as follows.

Definition*Let $\mathcal{H}^+$ denote the set of all pairs (φ, γ), where $\varphi \in C_b(\mathbb{R}_+; E)$ and $\gamma \in \mathbb{R}$ such that*

1. the function φ is given by

$$\varphi(t)(x,y) = \Phi(x,y,t), \tag{4}$$

where Φ is a perturbation of the form ,

$$\Phi(x,y,t) = Re\left(\widetilde{\Phi}(y)e^{i\alpha(x-ct)}\right), \qquad (x,y,t) \in [-R,R]\times[-1,1]\times\mathbb{R}_+,$$

$$\widetilde{\Phi}(y) = (\mathbf{u}(y), \mathbf{w}(y), \mathbf{p}(y))$$

which corresponds to certain values $\alpha > 0$, $c \in \mathbb{C}$, $Im\, c \leqslant 0$;

2. the perturbation Φ satisfies

$$|\Phi(x,y,t)| \leqslant R_0 \quad (x,y,t) \in \overline{\Omega} \times \mathbb{R}_+;$$

3. γ is given by the formula

$$\gamma = \begin{cases} (-\alpha Im\, c)^{-1}, & \textit{if } Im\, c < 0, \\ 0, & \textit{if } Im\, c = 0. \end{cases}$$

We say that $\mathcal{H}^+$ is the trajectory space *for considered system or the* perturbation space*.*

Evidently the pair $(0, 0)$, where the first zero denotes the zero perturbation, belongs to $\mathcal{H}^+$, thus the perturbation space is nonempty.

For the sake of convenience we shall proceed separately with the decaying perturbations and the periodic ones. By definition, we set

$$\mathcal{H}_1^+ = \{(\varphi, \gamma) \in \mathcal{H}^+ : \gamma > 0\};$$
$$\mathcal{H}_2^+ = \{(\varphi, \gamma) \in \mathcal{H}^+ : \gamma = 0\}.$$

We consider two kinds of attractors, namely phase and trajectory ones.

The *phase attractor* of the perturbation space is the minimal closed and bounded in E set that has the attraction property:

$$\sup_{(\varphi,\gamma)\in B} \inf_{f\in\mathcal{A}} \|\varphi(t) - f\|_E \to 0 \quad (t \to \infty)$$

for any $B \subset \mathcal{H}^+$ bounded in $C_b(\mathbb{R}_+; E) \times \mathbb{R}$.

The minimality means that the phase attractor is contained in any closed and bounded attracting set. Hence it is unique.

Note that the phase attractor is contained in the phase space. On the contrary, the *trajectory attractor* consists of functions of time taking values in the phase space. Each trajectory aquires resemblance to functions belonging to the trajectory attractor over time.

It is shown that in the model at issue the trajectory attractor is made up of families of homothetic ellipses (regarded as parametric curves) in a Hilbert space of vector functions. The phase attractor is the union of all these ellipses.

We use two approaches for visualization. First, we visualize the attractors by means of projecting the ellipses onto a three dimensional space. Next, we plot functions that characterize each ellipse, namely the v-component of its axes.

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