Collagen triple helix, discrete Hopf fibration, dislocations ...

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Abstract

Collagen has helicoidal structure on different scales: submolecular, molecular, fibrils, fibers ... It could be considered in twisted boundle cross-section containing rotational disclination screened by edge dislocations. Such frustration between 2D and chiral ordering is important for assemblies of biological polymers, from extra-cellular proteins like collagen and fibrin to cytoskeletal filaments, microfibrils, DNA-RNA, actin etc., organized into densely-packed states in cells and tissues. A collagen molecule (tropocollagen) consists of 3 polypeptide chains intertwined as in Boerdijk-Coxeter triple helix chain of tetrahedra. It is not periodic, because distances separating centres of neighbouring tetrahedra and pitch of three helices are not commensurable, causing frustration. It can be represented in 4D polytope (3, 3, 5), which is a discrete regular scaffolding of S3 hypersphere, made of 600 tetrahedra. 120 vertices of this polytope can be distributed by 10 in each, on 12 nonintersecting great circles of S3. Such great circles constitute fibers of discrete Hopf fibration. They are Clifford parallel, being at constant distance to each other. They are also entangled, winding once around each other. Presented simulated animation visualize glide and climb of edge dislocations occurring during conformational refolding of collagen on cylindrical deltahedra. Also on disk where collagen fibrils are close-packed in quasi-hexagonal phyllotaxis pattern. Similar by J.-F. Sadoc, N. Rivier, J. Charvolin, G.M. Grason ...

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