

On the dimensions of projective indecomposable modules for simple groups of Lie type

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Abstract

Let G be a finite group, p a prime dividing the order of G , and S a Sylow p -subgroup of G . Let F be an algebraically closed field of characteristic p .

A projective indecomposable module is defined to be an indecomposable FG -module whose restriction to S is a free FS -module. Projective indecomposable modules were introduced in 1940 by Brauer and Nesbitt. They play a significant role in modular representation theory of finite groups. Nonetheless very little is known about their dimensions. An obvious fact is that these are divisible by $|S|$, the order of S . When the dimension is equal to $|S|$? Surprisingly, this simple question has no simple answer, apart for p -solvable groups.

Brauer and Nesbitt established a natural bijection between projective indecomposable modules and irreducible FG -modules. The projective indecomposable module that corresponds to the trivial FG -module 1_G is called principal. In 2008 Malle and Weigel determined all principal projective indecomposable modules of degree $|S|$ for all simple group G and all prime divisors of $|G|$.

The following result contributes to the study of projective indecomposable modules of degree $|S|$:

Theorem. Let G be a simple group of Lie type in defining characteristic p , and S a Sylow p -subgroup of G . Let Φ be a projective indecomposable module of dimension $|S|$. Then either Φ is irreducible and hence the Steinberg module, or principal, and hence belongs to the list produced by Malle and Weigel.

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