Identities in varieties generated by algebras of subalgebras

Anna Zamojska-Dzienio A.Zamojska@elka.pw.edu.pl
Warsaw University of Technology, Poland
Coauthors: Agata Pilitowska

Abstract

In a natural way we can "lift" any operation defined on a set A to an operation on the set of all non-empty subsets of A and obtain from any algebra (A, Ω) its *power algebra* of subsets. Defined in such way *power* operation is a natural generalization of the multiplication of cosets of a subgroup of a group introduced by Frobenius. The same construction appears in the lattice of ideals of a distributive lattice and in formal language theory where the product of two languages is the power operation of concatenation of words. Some properties of an algebra (A, Ω) may remain invariant under power construction but obviously not all of them. In particular, not all identities true in (A, Ω) will be satisfied in its power algebra. G. Grätzer and H. Lakser proved that for a variety $\mathcal{V}$, the variety $\mathcal{V}\Sigma$ generated by power algebras of algebras in $\mathcal{V}$ satisfies precisely the consequences of the linear identities true in $\mathcal{V}$. For certain types of algebras, the sets of their subalgebras form subalgebras of their power algebras. They are called *the algebras of subalgebras*. We partially solve a long-standing problem concerning identities satisfied by the variety $\mathcal{VS}$ generated by algebras of subalgebras of algebras in a given variety $\mathcal{V}$. We prove that if a variety $\mathcal{V}$ consists of algebras which are idempotent (each singleton is a subalgebra) and entropic (each two operations commute) and the variety $\mathcal{V}\Sigma$ is locally finite then the variety $\mathcal{VS}$ is defined by the idempotent and the linear identities true in $\mathcal{V}$.

AMS Classification: Primary 03C05; Secondary 08A30.