

Transfinite extension of the asymptotic dimension related to the finite decomposition complexity

M. Zarichnyi

(Institute of Mathematics, University of Rzeszów)

E-mail address: zarichnyi@yahoo.com

Recall that a metric space X has the asymptotic Property C if for every $R_1 < R_2 < \dots$ there exist $n \in \mathbb{N}$ and uniformly bounded R_i -disjoint families \mathcal{U}_i , $1 \leq i \leq n$, such that $\cup_{i=1}^n \mathcal{U}_i$ covers X (see [1]). T. Radul [3] characterized the spaces with the asymptotic Property C as those for which the transfinite asymptotic dimension asdim is defined.

The finite decomposition complexity property (FDC) of metric spaces is defined in [2]. This is a large scale property of a metric spaces. It generalizes finite asymptotic dimension and also applies to investigations of groups. some groups. In particular, it is shown that Thompson's group does not have the FDC (see [4]).

We use a version of the FDC property to obtain another transfinite extension of the asymptotic dimension. Some properties, e.g., coarse invariance and the sum theorem, of the introduced transfinite asymptotic dimension are established. In particular, the class of spaces for which this dimension is defined is strictly larger than that having the asymptotic Property C.

Some open questions are formulated.

References

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