Bernoulli numbers, Drinfeld associators and the Kashiwara-Vergne problem

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Abstract

In 1978, Kashiwara and Vergne put forward a conjecture on the properties of the Campbell-Hausdorff series. Among other things, the Kashiwara-Vergne conjecture provides a natural (independent of structure theory) proof of the Duflo isomorphism between the center of the universal enveloping algebra and the ring of invariant polynomials. The conjecture is given by two linear equations for a pair of Lie series, $a(x,y)$ and $b(x,y)$. The first equation is of the form

$$\ln\left(e^x e^y\right) = x + y + [x, a(x,y)] + [y, b(x,y)],$$

and the second equation is expressed in terms of the generating function of Bernoulli numbers $x/(e^x-1) = \sum_{k=0}^{\infty} B_k x^k/k!$. There is another object in Lie theory which makes use of Bernoulli numbers. This is a Drinfeld associator satisfying the pentagon equation. In the talk, we explain how one can prove the KV conjecture using Drinfeld associators. We also state several conjectures arising from comparison of topological, number theoretic and Lie theoretic approaches to associators.

The talk is based on joint works with B. Enriquez, E. Meinrenken and C. Torossian.

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