Holomorphic deformations, quasiconformal mappings and vector valued calculus of variations

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Abstract

In this talk, based on joint work with T. Iwaniec, I. Prause and E. Saksman, we describe a new approach to vector valued calculus of variations, via holomorphic deformations and ideas from quasiconformal mappings.

A fundamental question in the calculus of variations is to characterize quasiconvex functionals $F : \mathbb{R}^{n\times m} \to \mathbb{R}$, equivalently, functionals for which the energy integral $\int F(Du)dx$ is lower semicontinuous. Somewhat surprisingly, the vector valued case $m, n \geq 2$ is still widely open. According to Morrey, the convexity of F in the direction of rank-one matrices is a necessary condition, but as shown by Sverk, not sufficient in dimensions $m \geq 3$. On the other hand, there is an abundace of evidence for sufficiency in planar domains, $n = m = 2$.

The talk will focus on an important functional originated in the work of Burkholder on optimal martingale inequalities. The conjectural quasiconvexity of the Burkholder functional motivates several deep results e.g. in singular integrals and PDE's.