Old and new results on biharmonic submanifolds in spheres

Adina Balmuş adina.balmus@uaic.ro
Al.I. Cuza University of Iasi, Romania
Coauthors: Stefano Montaldo and Cezar Oniciuc

Abstract

A *biharmonic map* $\varphi : (M, g) \to (N, h)$ between two Riemannian manifolds is a critical point of the bienergy functional

$$E_2(\varphi) = \frac{1}{2} \int_M |\tau(\varphi)|^2 \, v_g,$$

where $\tau(\varphi) = \operatorname{trace} \nabla d\varphi$ is the *tension field* of φ. The notion was proposed by Eells and Lemaire as a natural generalization of harmonic maps ($\tau(\varphi) = 0$). Although introduced in the 1960's and studied from the analytical point of view, biharmonic maps have become a field of study for geometers relatively recently. Our attention was focused on *proper biharmonic submanifolds*, i.e. on biharmonic non-harmonic (non-minimal) Riemannian immersions, and the first ambient spaces to be considered were the space forms. Due to some non-existence results for proper biharmonic submanifolds in space forms of non-positive sectional curvature, our study concerns the unit Euclidean sphere $\mathbb{S}^n$.

The present poster surveys on old and new classification results for proper biharmonic submanifolds in $\mathbb{S}^n$, taking mainly into account the hypersurfaces and the submanifolds with parallel mean curvature vector field.

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